THE EFFECTS OF MATHEMATICS THEIR WAY AND CHICAGO MATH PROJECT ON MATHEMATICAL APPLICATIONS AND STORY PROBLEM STRATEGIES OF SECOND GRADERS

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The problem. The purpose of this study was to address two different manipulative approaches in the second grade mathematics curriculum by comparing them to a traditional delivery using a textbook. Story problem/application achievement scores were compared to see if one of the new manipulative approaches would better meet the needs of students in compliance with the Curriculum and Evaluation Standards for School Mathematics written by the National Council of Teachers of Mathematics (1989).

Procedures. A sample of 250 second graders were taught one of three mathematics treatments. The three treatments included Treatment T, using the traditional paper-and-pencil computation method; Treatment M, Mathematics Their Way and teacher-designed worksheets for specific outcomes; and Treatment C, University of Chicago Mathematics Project. Before and after 27 weeks of instruction students were given a test on problem solving and application. An ANCOVA was applied in order to adjust any differences in groups. The students' pretest score, I.Q., and age were used as covariates in order to find any statistically significant differences in any one of the three mathematical treatments. A Delta formula was applied for any differences in effect size.

Findings. There was no statistically significant difference in any one of the three mathematical treatments when pretest and age were controlled, when pretest and I.Q. were controlled, or when pretest, age, and I.Q. were controlled. A Delta formula indicated a 16% gain made by treatment M students over Treatment T students. The researcher would caution the reader, however, that the pretest, age, and I.Q. were not controlled in the Delta results.

Conclusions. The methodology for instruction seemed unimportant when all teachers taught to the same outcomes, supporting the philosophy of Outcome Based Education. Teaching to specific outcomes is probably as effective, if not more effective, than teaching with a "set" textbook curriculum in the second grade. The researcher found that the teacher has more to do with achievement than specific curricula. Increased structure and direction had a positive effect on all treatments.

Recommendations. Due to the support of Outcome Based Education, the study has implications for a K-4 curriculum. Teachers need to facilitate learning mathematics by using a variety of instructional resources including manipulatives. The researcher would recommend staff development training in using manipulatives and teaching to outcomes in lieu of expensive textbooks/workbooks for primary grade levels. Further research is recommended in intermediate grade levels with a mixture of experienced and non-experienced teachers. Further research is also needed using a dependent variable other than a standardized test.

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Chapter 1

INTRODUCTION

Rationale

American mathematics students are falling behind their Japanese and Chinese counterparts in mathematics achievement (Stevenson, Lummis, Lee, & Stigler, 1990; Stigler, Lee, & Stevenson, 1990). According to the Stevenson et al. and Stigler et al. studies, mediocre performance was noted among American elementary children as compared to Asian elementary children. Statistically significant differences were recorded in favor of Asian first and fifth-grade children in both computational and problem-solving items.

The Second International Mathematics Study illustrated poor performance of United States students compared to their international peers (National Council of Teachers of Mathematics, Curriculum and Evaluation Standards for School Mathematics [hereinafter referred to as NCTM, 1989]). For example, in arithmetic, eighth graders scores 10th out of 20, and twelfth graders 12th out of 15 (NCTM, 1989).

Even when the top 5% of twelfth-grade United States (U.S.) students were compared to their peers in other nations, United States students ranked 15th out of 15. United States students also scored near the bottom when 13-year-old children from five countries and four Canadian provinces competed in A World of Differences: An International Assessment of Mathematics and Sciences (NCTM, 1989).

The results of U.S. students' performance should not reflect only on schools and teachers but on the failure of society to react to demands of world competition. Student involvement and achievement in challenging subject matter should be promoted and valued. A change in education can be accomplished by admitting that a problem exists (Minnesota Department of Education, 1990c).

The Task Force on Mathematics, Science, Technology, and International Education formed by Perpich, Governor of Minnesota, has made several recommendations asking educators to cooperate in their vision for tomorrow. They recommended that student involvement in more challenging subject matter be promoted and valued, along with strategies for involving minorities and females in mathematical academic work (Minnesota Department of Education, 1990c).

Minnesota students are among the two groups in the U.S. who did not perform well in mathematics when compared to the achievement levels of students from Asian countries. The Governor's Task Force has suggested that leaders at all levels must support change. The research in Minneapolis, Minnesota, exhibited large differences in mathematical achievement and yet there was no overall difference in intelligence as measured by a cognitive abilities test (Stevenson et al., 1990).

Deficits in performance are being taken seriously because achievement in mathematics is fundamental to our economy, national security, and high standard of living (Stevenson et al., 1990). The improvement of mathematics education has become a national concern. Considering that the current curriculum is a product of the 19th century industrial age, the National Council of Teachers of Mathematics (NCTM) has set new standards for the 21st century in order to promote change and ensure quality. They have also set new guidelines for mathematics instruction (NCTM, 1989).

The National Assessment of Educational Progress found that while American students' basic skills have actually improved over the last 20 years, few students can apply what they know to problems outside simple computation (NCTM, 1989). For example, in the same study, nearly one-half of all 17-year-old children could not say whether 87% of 10 was greater or less than 10 (NCTM, 1989).

Everybody Counts: A Report to the Nation on the Future of Mathematics Education (National Research Council, 1989), and Reshaping School Mathematics (Mathematical Sciences Education Board, 1990) have provided a rationale for change that will be discussed later in the report. The NCTM

Standards (1989) have suggested a "vision" for restructuring the K-12 mathematics programs in today's schools.

Challenge 2000: Success for All Learners has also suggested goals for restructuring the kindergarten through twelfth grade (K-12) curriculum. One of the goals recently suggested was that schools assist students in developing skills for lifelong learning (Minnesota Department of Education, 1990a). This same goal was listed in NCTM (1989). More specifically, the objective under that goal was to treat kindergarten through third grade (K-3) as a learning block for helping children to achieve the basic skills. More opportunities would exist for developing a curriculum appropriate to the child's age and life experiences. More questions need to be answered concerning the restructuring of the K-3 mathematics program in order to meet the new goals of the 21st century.

The Problem

A change and restructuring of the mathematics program in compliance with NCTM (1989) is needed. Using the Outcome Based Education (Spady, 1991) philosophy, students need to have met certain outcomes before they reach high school. A solid foundation in the early grade levels is a necessity in order to meet outcomes that were designed for the high school grade levels and for graduation exit outcomes.

The researcher will focus on the second-grade mathematics program because outcomes are not being met in the second grade level according to primary teachers in the Farmington School District. Mathematics Their Way (MTW) has been the methodology in the kindergarten and first grade levels. MTW was selected and implemented for the primary grade levels because of the research supporting Piagetian philosophy and the use of manipulatives for teaching on the conceptual level (Carpenter, Fennema, & Peterson, 1988; Parham, 1983; Perry & Grossnickle, 1987; Suydam & Higgins, 1977).

Second and third grade teachers have noticed, however, that children are farther behind on the Comprehensive Test of Basic Skills than previous students who were taught with a basal textbook mathematics approach. The primary teachers in the Farmington District have legitimate concerns in regard to the delivery system inasmuch as their comments are supported by the research on first and fifth grade children (Stevenson et al., 1990; Stigler et al., 1990). The study revealed that American children lag behind Asian counterparts in mathematical achievement as early as the first grade. The difference becomes more pronounced by the fifth grade (Stevenson et al., 1990; Stigler et al., 1990).

A review of the literature indicates that demonstrable mathematics outcomes are important (Spady, 1991). In other words, the emphasis is placed on what the learner can do and apply, not on what the textbook covers and what the student can memorize. The teaching style and underlying belief that all young children can learn in a whole group setting is as important as the specific curricula (Stevenson et al., 1990). Asian children were expected to learn regardless of intelligence quotient (I.Q.) or developmental readiness (Stevenson et al., 1990). For example, special education is not provided in public schools in Japan.

Using a curriculum that is designed to clearly focus on desired demonstrable outcomes appears to be a problem in American schools (Stevenson et al., 1990). The chief variation in mathematics classes was not primarily the content of the textbook, but how well the curriculum was taught and the degree to which the goals of the curriculum were met (Stevenson et al., 1990).

Restructuring recommendations made by the NCTM in response to poor performance of American elementary children, compared to Asian children, included a change in methodology, specific guidelines, parental involvement, and a new belief system that embodies the notion that all students can learn (Stevenson et al., 1990; Stigler et al., 1990).

Our nation's previous excessive emphasis on the mechanics of mathematics has inhibited learning (National

Research Council, 1989). In the past, parents, community members, and businesses expected accountability which was demonstrated in most Minnesota districts by standardized norm-referenced tests. State mandated Planning Evaluating and Reporting (PER) legislation in Minnesota has required school districts to plan, evaluate, and report test scores and provide a plan for improvement at the end of each school year. Norm-referenced tests, when used as the outcome, reinforced the message that the only problems amenable to mathematics were those that had correct answers. The "product" was considered important and the "process" used to formulate the answer was ignored. When the outcome embraced the old "product" philosophy, as in the past, schools and administrators have resorted to "product" oriented curriculum. For example, the concentration was on getting the right answer as opposed to exploring multiple strategies or processes for different ways of getting the answer. In the past century, schools have concentrated on this "product" or answer-based curricula that were easily assessed by paper and pencil tests. A set of specific mathematics guidelines recommended by the NCTM would be synonymous with "clearly defined outcomes" which have been mandated in Minnesota some time before 1996 (Minnesota Department of Education, 1991).

Likewise, Outcome Based Education (OBE) has given a purpose to restructure, that is, a range of beliefs, conditions, practices, and traditions to attain a specific end: All students can succeed and schools control the conditions of success (Minnesota Department of Education, 1990b; Spady, 1990).

Stories of Excellence: Ten Case Studies from a Study of Exemplary Mathematics Programs described an exemplary school. At Dawson Elementary School in Ann Arbor, Michigan, teachers developed their own curriculum by using several resource programs (Driscoll, 1987). Mathematics Their Way, ABC Mathematics Assessment Project (Mississippi Bend Area Education Association, 1986), the Addison-Wesly basal manual, and teacher-made materials were among the resources used. It would appear that a "blend" of process and product may be what is best for the learner. The "blend" will depend on the methods that meet the criteria of the mathematics standards in problem solving and the specific outcomes for each grade level. The dilemma of new methodology placing more emphasis on "process" and assessing that process must be addressed. The problem is knowing which process and methodology makes the best use of mathematical instruction in a meaningful way.

Importance of the Study

Mathematics is of particular interest because it is a discipline about which educators and parents throughout the nation are expressing a growing concern. One of the alarming studies in Minnesota was completed in Minneapolis (Stevenson et al, 1990). The Stevenson et al. (1990) and Stigler et al. (1990) studies will have an impact on the type of restructuring that schools will undertake and the teaching methods used. In order to ensure the conditions of success for all primary children, new methodologies that are based on current research must be considered. More specifically, the research will be helpful to districts because of differing opinions by staff members. Some teachers, for instance, are very textbook oriented and want to continue a very structured approach. This approach is an example of a product-oriented curriculum whereby the main emphasis is on memorization of mathematical facts and on computation. Students are presented with drill and practice worksheets following every lecture. Other teachers are using manipulatives for discovery learning by allowing students to discuss multiple ways of reaching the answer. The latter is a process approach involving more active participation by the learner and less lecturing by the teacher.

The "product oriented" basal program used in the past century was governed by a rigid belief system dictated by rules, accuracy, speed, and memory. Further analysis is needed in order to find a "blended" approach in alignment with the NCTM Standards, goals of the Minnesota State Education Department, and the developmental level of the child (Minnesota Department of Education, 1990a).

Schools need to overcome the barriers to reforming and restructuring. New content and methods in the past rarely became standard practice because the philosophy stayed the same. The target statement for curriculum directors and administrators in charge of mathematics programs has changed. Self-confidence is an important objective in the mathematics curriculum for building success (National Research Council, 1989).

State education departments and schools need to recognize the limitations of mathematical models and resort to a balance of the two philosophies for meeting the new standards. A "blended" methodology will require a tight alignment between written outcomes, actual instruction, and assessment. Additionally, the methodology needs to target student self-confidence by resorting to developmentally appropriate practices. The research on the way children learn and what will best fit the needs of young children are important considerations for a product-oriented program.

Purpose of the Study

The purpose of the study is threefold:

1. To determine the relative effectiveness of mathematics programs that incorporate different ways of using manipulatives in order to note any statistical significance;
2. To determine if there is a difference in achievement between three methodological treatments on second graders' mathematical applications and story problem-solving strategies using addition and subtraction;
3. To consider the findings of this study and make a recommendation on which second-grade methodology should be used in District #192 for the 1992-93 school year.

District #192 uses Mathematics Their Way (MTW) in kindergarten and first grade, an older version of the Scott Foresman (1990) basal text in third through sixth grade, and the University of Chicago School of Mathematics Project (UCSMP, 1991) in seventh through twelfth grade.

Mathematics Their Way (MTW) is a program designed only for primary students. It is recommended for K-3 grade levels (Baratta-Lorton, 1976).

University of Chicago School Mathematics Project (UCSMP) was made available for second grade on August 1, 1991, for the 1991-92 school year. Third through sixth-grade programs are to be written in the near future.

Definition of Terms

Analysis of Covariance (ANCOVA) is similar to an analysis of variance. The ANCOVA is used to determine whether mean scores on one or more factors differ significantly from each other with the influence of one or more independent variables on the controlled dependent variable. The statistical technique is used to control for initial differences between groups. The effect is to establish "equivalent" groups with respect to one or more control variables when random assignment is impossible (Borg & Gall, 1989; Ferguson & Takane, 1989).

California Diagnostic Mathematics Test is an assessment providing norm-referenced information that is not only valid but also reliable for all students, including those who test below the 50th percentile (California Diagnostic Mathematics Test, 1989). Sample questions may be found in Appendix A.

Level B, which will be used in this study, is intended for a second to third grade range. The applications portion contains questions on modeling word problems, solving word problems, measurement and geometry, and graphing.

Learner Outcome is that bank of knowledge which a student must be able to demonstrate in order to advance to the next unit, level, or course.

Mathematics Their Way (MTW) is a mathematics program written for kindergarten, first, second, and third graders based on a philosophy of discovery learning by using manipulatives (i.e., concrete objects for understanding concepts before working with symbols). MTW is the manipulative approach used in Treatment M of this study. Students use concrete objects with three levels of instruction including concept, connecting, and symbolic instruction (Baratta-Lorton, 1976). Examples of the different instructional levels are in Appendix B.

Manipulatives, or hands-on materials, are concrete objects that can be manipulated to illustrate the concept and can be experienced visually by the child. They are used extensively in Mathematics Their Way and University of Chicago School Mathematics Project (1991). Counters are used with some of the worksheets in the Scott Foresman (1990) Exploring Mathematics textbook. The difference is the way in which they are used. A variety of familiar materials are used by the child to build a bridge to the adult world of abstraction (Baratta-Lorton, 1976).

Multiple Strategies refers to solving a mathematical problem using more than one solution for the answer (NCTM, 1989).

The NCTM Standards are the curriculum guidelines that describe criteria for a quality mathematics curriculum from kindergarten through the twelfth grade. The guidelines include a greater emphasis on conceptual development, mathematical reasoning, and problem solving (NCTM, 1989).

This study will focus on the K-4 standards that emphasize word problems with a variety of structures, problem-solving applications, patterns and relationships, geometry and measurement, thinking strategies for basic facts, and the use of calculators for complex computation (NCTM, 1989). The K-4 NCTM Standards are found in Appendix C.

Outcome Based Education (OBE) is a way of designing, developing, delivering, and documenting instruction in terms of its intended goals and outcomes (Spady, 1991). An example of second grade mathematics outcomes is found in Appendix D.

Minnesota OBE is defined as education programs designed and implemented in a manner that assures alignment of three basic elements (i.e., [a] learner outcomes, [b] assessment and feedback process, and [c] instructional practice) (Minnesota Department of Education, 1990b). This philosophy is embraced by the state of Minnesota. School districts are required to have mathematics learner outcomes according to Minnesota Rule 3500.1075 Supp.6,A-J (Minnesota Department of Education, 1991).

Stage of Concrete Operations (age 7-11) is the stage according to Jean Piaget (1952) where a child's reasoning processes become logical. During this stage, the child evolves logical thought processes or operations that can be applied to concrete problems (Piaget, 1967; Wadsworth, 1989).

Test of Cognitive Skills (TCS) is a reliable and valid academic aptitude test that measures the three critical cognitive factors (i.e., verbal, non-verbal, and memory). The TCS yields a score that indicates a student's overall academic aptitude with the same statistical properties as an Intelligence Quotient (Test of Cognitive Skills Norms Book, 1982). The Kuder-Richardson formula 20 (KR 20) was applied to TCS for reliability (Test of Cognitive Skills Technical Report, 1982).

Traditional Mathematics is any basal mathematics program written in a textbook format for several grade levels. The traditional approach concentrates on paper-pencil computation and rote memory of mathematical facts. Exploring Mathematics by Scott Foresman (1990) is a traditional basal textbook used for Treatment T in this study. Examples are found in Appendix E which are representative of many lessons taught on the symbolic level (Exploring Mathematics, 1990).

University of Chicago School Mathematics Project (UCSMP) (1991) is currently a 7-12 mathematics program. The K-6 elementary portion is in the process of being written and field studied at each grade level. Kindergarten and first-grade programs have been written and the second-grade program was completed on August 1, 1991.

The UCSMP (1991) for second graders is entitled Second Grade Everyday Mathematics and is Treatment C in this study. The manipulative program consists of teaching techniques that involve different instructional levels as in MTW. Calculators are used in this methodology.

The format is designed to help the student make transitions between concrete, pictorial, and symbolic representations (University of Chicago School Mathematics Project, 1991). Examples are shown in Appendix F.

Null Hypotheses

The investigator will use the following null hypotheses for the study:

1. There is no statistically significant difference in the posttest score means on a mathematics application assessment between a traditional textbook approach and other manipulative approaches when the pretest and I.Q. are controlled.
2. There is no statistically significant difference in the posttest score means on a mathematics application assessment between Mathematics Their Way and University of Chicago School Mathematics Project (1991) when the pretest and I.Q. are controlled.
3. There is no statistically significant difference in the posttest score means on a mathematics application assessment between a traditional textbook approach and other manipulative approaches when the pretest and age are controlled.
4. There is no statistically significant difference in the posttest score means on a mathematics applications assessment between Mathematics Their Way and University of Chicago School Mathematics Project (1991) when the pretest and age are controlled.

5. There is no statistically significant difference in the posttest score means on a mathematics applications assessment between a traditional textbook approach and other manipulative approaches when the pretest, I.Q., and age are controlled.
6. There is no statistically significant difference in the posttest score means on a mathematics applications assessment between Mathematics Their Way and University of Chicago School Mathematics Project (1991) when the pretest, age, and I.Q. are controlled.

Chapter 2

REVIEW OF THE LITERATURE

Introduction

The National Council of Supervisors of Mathematics (NCSM) and the National Council of Teachers of Mathematics (NCTM) have continued to search for ways to effect needed curricular changes. The NCSM's Essential Mathematics for the Twenty-first Century (NCSM, 1988) was created for the purpose of a reform in mathematics education. "Essential mathematics" as described in this paper represents the mathematical competence required for responsible adulthood. Students today who will be adults in the 21st century will change jobs often in their lifetimes (NCSM, 1988). Skill in whole-number computation has been outdated as an adequate indicator of mathematics achievement. Instead, students will need the following in preparing for mobility:

1. To understand mathematical concepts and principles
2. To reason by using effective communication skills
3. To recognize mathematical applications in the world around them
4. To become confident in approaching mathematics problems
5. To apply fundamental skills in new situations
6. To control their own lifelong learning (NCSM, 1989)

The NCTM has indicated five new goals for K-12 students:

1. To learn to value mathematics
2. To become confident in personal ability to do mathematics
3. To become mathematical problem solvers
4. To learn to communicate mathematically
5. To learn to reason mathematically (NCTM, 1989).

The K-4 standards are noteworthy because they have implications in the second-grade curriculum. More specifically, in grades K-4, the mathematics curriculum should include a study of patterns and relationships so that students are able to:

1. recognize, describe, extend, and create a wide variety of patterns
2. represent and describe mathematical relationships
3. explore the use of variables and open sentences to express relationships (NCTM, 1989).

Activities mentioned in the K-4 NCTM Standards would provide opportunities for primary children to become active participants in creating knowledge instead of passive receivers of rules and procedures.

The NCSM is in agreement with NCTM on changing the methods of delivery according to the Professional Standards for Teaching Mathematics. Mathematics textbooks in the past dealt with the development of skills apart from applications. Memorization of facts and paper-pencil computation have not satisfied the NCTM requirements. Mathematics teachers have been concerned about ways to develop proficiency in problem solving and higher-order thinking skills. The NCTM has agreed that a curriculum revision is imperative if schools plan to incorporate the 13 K-4 critical areas of mathematical competence for all students as explained in Appendix C. These are:

Mathematics as problem solving

Mathematics as communication

Mathematics as reasoning

Mathematical connections

Estimation

Number sense and numeration

Concepts of whole number operations

Whole number computation

Geometry and spatial sense

Measurement

Statistics and probability

Fractions and decimals

Patterns and relationships (NCTM, 1989)

At the K-4 level, the NCTM's Standards (1989) focus on regularity in events, shapes, designs, and sets of numbers. Young students need encouragement in order to recognize, extend, describe, and create a wide variety of patterns. This would include identification and description by using open sentences to express the relationships. The young student will become involved in a verbal expression of relationships or descriptions following identification (NCTM, 1989).

Learning Theories in Relation to Mathematics Instruction

The rationale for using manipulative materials in the classroom has been implied in Jean Piaget's (1952) theories of cognitive development. Many K-2 programs, however, have not allowed the time or opportunity for development of number relations as described in the Piagetian theory (Wadsworth, 1989).

Jean Piaget believed that children had ways of teaching teachers. His studies acknowledged that children's thinking processes were different from adults (Piaget, 1971, 1973).

According to Piagetian methodology, the child was presented with objects from the environment while the interviewer observed and listened to what the child said in response to the materials.

Additionally, the Piagetian viewpoint indicated that student knowledge was not simply absorbed passively. On the contrary, Piaget believed that knowledge was constructed by the child through interactions between his/her mental structure and the environment. A child reached the stage of concrete operations between the ages of 7 and 11. Some children entered the concrete operations stage at 5 while others entered the stage at 9. All children passed through a concrete operational stage but at various rates.

The concrete operational child was found to be superior to the preoperational child because he/she improved in understanding of concepts of causality, space, time, and speed. A functional use of logic was not evidenced in the behavior of children younger than seven. Operations such as reversibility and classification were useful in solving problems using concrete objects or events in the immediate present. Concrete operational children were not yet able to attack hypothetical or abstract problems with any logic. If concrete operational children were presented with purely verbal problems, they were usually unable to solve them. When the same problem was presented with real objects, they could solve the problem. The concrete operational stage was viewed as a transition between the preoperational (prelogical) and the completely logical thought of older children (Wadsworth, 1989).

The stage of formal operations began around the age of 11 or 12. The child developed the reasoning and logic to solve all classes of problems. Children then had the potential of "adult" thought. The quality of reasoning was less mature and the content improved beyond this stage into adulthood.

Each new level of reasoning was a refinement of a prior level of reasoning and as such was not totally new. With each stage of development there was an integration and extension of the knowledge and reasoning of the previous level into "new" knowledge. Piaget recognized the "right" experience at the "right" time (Wadsworth, 1989). In other words, children need developmentally appropriate instruction in the concrete operational stage.

Differences in prior experiences contributed to "individual differences" in cognitive development. The more children experienced physical objects in their environment, the more they developed a related understanding. Teaching by telling proved meaningless when conducted in the absence of direct experience with the objects. It was found that children obtained physical knowledge as well as logical knowledge by manipulating objects (Wadsworth, 1989).

Implication of Piaget's Theory for Mathematics Curricula

Piaget insisted that words and symbols could serve as useful labels or reminders only after the child had constructed the relationship through his/her own experience with objects. Knowledge was not derived from the objects but from manipulation of the objects.

There was nothing in Piaget's theory that conflicted with educational goals in the United States. However, there was a conflict with the way schools had reached those goals. Educators who believe in Piaget's theory have used "development" in their goals for education. This goal did not imply that traditional skills were eliminated. The belief was that in a school system that had been organized to encourage development, skills and knowledge will be acquired more effectively than in schools with conventional organization.

From the Piagetian perspective, rote memory was not part of intellectual development. This did not mean that rote learning was not valued. This simply meant that memorization did not imply comprehension. Children who comprehended mathematical operations were intellectually different from those who had only memorized computational procedures.

The most important implication of Piaget's theory was the realization that children construct knowledge from their actions on the environment. If the objective in education was to enhance children's knowledge, educational methods needed to be consistent with this objective.

The role of the teacher was central in a Piagetian classroom. Teachers spent less time lecturing, and less time on work sheets. The teacher assessed the level of readiness for each individual. There were three ways suggested to assess the level of cognitive development: (a) Testing, using Piagetian procedures, (b) Observing child's reasoning, and (c) Watching for "spontaneous interests" (Wadsworth, 1989).

The Piagetian theory has been misinterpreted as having "all the answers" (Wadsworth, 1989). Rather, it validated a belief that understanding children was the best way to improve curriculum and teaching. Instead of focusing on acceleration in the early years, Piaget intended that teachers provide opportunities to explore and build the strongest possible foundation for succeeding stages. He also noted that the attributes critical in facilitating children's thinking were the same as the attributes of good teaching.

Wadsworth (1989) stated that traditional methods in mathematics may have a detrimental effect on children's learning. He thought bright students were more or less permanently handicapped by nonconstructive teaching methods and mathematics curricula. The main culprits were methods that focused on correct answers rather than on thinking and constructing.

Wadsworth's (1989) contention, based on Piaget's theory, was that learning mathematical concepts and procedures required the application of concrete and formal operations to the content. New or different forms of reasoning were not required. There was no special mathematical aptitude. Those who understood mathematics constructed concepts out of their logical-mathematical reasoning, independent of the instruction.

Role of Outcome Based Education on Mathematics Instruction

According to several studies, Outcome Based Education (OBE) has improved children's achievement in mathematics (Deever, 1991, 1992; Mamary, 1990; Vickery, 1990). The Outcomes-Driven Developmental Model in Johnson City, New York, and the Outcome Based Instruction Model in Glendale, Arizona, use pretests to determine entry level knowledge. Additionally, they provide whole-group instruction without tracking through cooperative learning. The cooperative learning techniques used by OBE teachers embrace the Piagetian philosophy of observing students' reasoning and interests before approaching instruction. Like Piaget, OBE teachers hold the belief that all students can learn mathematics concepts. Special mathematical aptitude is not a consideration. The NCTM Standards (1989) reflect the results of mathematics research and the new beliefs of OBE.

The analysis in the Japanese, Chinese, and American 1990 research show that the curricula did not differ greatly in the three countries (Stevenson et al., 1990; Stigler et al., 1990). The variation, however, in the United States' mathematics classes was not the content of the curriculum but the teaching strategies and the degree to which the goals of the curriculum were met. Methodology in Japan and China was more outcome based and students were expected to master the outcomes.

Emphasis on "product" in the United States' classroom may be a contributing factor to the poor mathematics performance of American elementary children when compared with Asian children (Stevenson et al., 1990).

Purposeful and Meaningful Instruction

The NCTM goals in the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) have reflected a need for meaningful instruction that provides understanding and reasoning skills on a higher cognitive level. Likewise, M. C. Wittrock's generative theory of learning maintained that learners needed to generate meaning themselves and that no one could do it for them (Wittrock, 1986).

Success in computation has often masked the failure of educators. Some elementary children spend an estimated 90% of their school mathematics time on paper-and-pencil computation practice (Stevenson et al., 1990). Most often young children learned computation skills by rote memorization through drill and practice.

The ability to count with understanding was an important problem-solving skill for kindergartners and first graders because it was the basis for finding solutions in addition and subtraction. The goal of kindergarten and first-grade teachers was to ensure that counting skills be intact before children were asked to apply the numbers in addition or subtraction facts (Thorton, 1978).

Questions that have arisen for educators from the Japanese, Chinese, and American study of first and fifth graders (Stevenson et al., 1990; Stigler et al., 1990) have been addressed with the suggestion that educators challenge students in the development of higher-level cognitive skills and understandings. Madeline Hunter's research indicates a need for meaningful instruction by teaching to an objective (Hunter, 1980b). The OBE philosophy suggests a clear focus by teaching to outcomes and deleting unnecessary trivia (Spady, 1991). This component was missing in American schools according to Stevenson et al. (1990). They concluded that American schools have no standard curriculum or clear definition of what children should learn in each grade.

Meaningful instruction was noteworthy for Japanese elementary children. It appeared that their teachers stressed the importance of thinking and evaluating alternative strategies for solving problems. This resulted in significantly higher mathematics scores (Stevenson et al., 1990; Stigler et al., 1990). The Japanese also deemphasized the speed of performance. Instead, there was a much stronger emphasis on thoughtful reflection and the importance of executing procedures carefully and correctly. The Japanese obtained the highest overall scores on the computation test but also attempted the least problems. The Chicago first graders were correct on 61% of the computation problems, whereas the Japanese solved 85% correctly. In fifth grade the pattern was similar except both Chinese and Japanese students solved 77% of attempted problems as compared to 51% of American children (Stevenson et al., 1990).

Japanese students consistently took the most time to complete the tasks. There was no area in which American children were competitive with the children from Japan and Taiwan, including computation, speed, or application of mathematical principles (Stevenson et al., 1990).

In the United States, sophisticated mathematical concepts were introduced but often treated in an abbreviated way. For example, probability was introduced in the fourth grade in the United States. Probability was introduced to sixth graders in Japan but inferential skills were immediately discussed. American children are not mastering underlying concepts and applying them accurately (Stevenson et al., 1990).

American and Chinese children tended to compute first and think later. Japanese students were taught differently and the effects showed up in their responses to oral problems. Data indicated that stereotyped word problems were taught in the United States classroom. Japanese children were more likely to evaluate whether or not they could make a correct reply (Stevenson et al., 1990). For example, Asian children were asked to make real-world associations with arithmetic operations. First and fifth grade students were given a simple equation such as 5 + 2 = ? and asked to make up a word problem to fit the equation. Seventy-nine percent of the Japanese responded with a valid story problem compared to 39% of Chinese and 44% of American first graders. By fifth grade, the Japanese percentage rose to 86%, Chinese to 85%, and American to 60% (Stigler et al., 1990).

Taiwanese and Japanese teachers were expected to elaborate on every lesson by supplementing the textbook with interesting discussion, creative examples, and relevant practice exercises. Their lessons were more likely to be standardized because teachers shared experiences and benefits from each other's successes and problems to a much greater degree than is possible in the United States

(Stevenson et al., 1990). Additionally, they used a ministry-defined curricula that was uniform across all classrooms; whereas in the United States, curricula differed among schools and among classrooms within the same school (Stevenson et al., 1990).

In order to comply with the new recommendations of NCTM and NCSM, schools will need to alter K-3 programs in which children associate meaning with an operation or a relationship before memorizing the fact. The drill is effective only when children put meaning or value to the mathematical operation first (Hunter, 1980b). When practice comes after meaning, however, the learning is faster and more permanent; less practice is needed than was necessary for rote learning (Hunter, 1992).

Other researchers have expressed concerns about a lack of understanding due to memorizing facts. Practice can be intelligently used only when meaning has played a part in the learning (Carpenter et al., 1988).

Madeline Hunter (1980b) stated that students needed to know the number facts in order to estimate the correctness of answers and to do simple problems in their heads. Her instructional strategy included the following steps: (a) the anticipatory set includes a meaningful example from the student's own life (i.e., from the student's experience rather than the teacher's experience); (b) a short practice session with only three to five unknown facts at a time was the best; (c) "distributed practice" where the teacher occasionally reinforces the learned facts; and (d) an individual chart to assist the teacher in record keeping and provide an indicator for the student of his/her own progress. The steps helped to encourage students' "self-practice" so they would use routine nonthinking times to do their own "distributed practice." When the students assumed responsibility they were not resentful about drill. Students needed to be able to diagnose what they understood and what they did not. This made them responsible for their own mathematics literacy (Hunter, 1980b).

Connecting Symbols with Understanding and Multiple Calculation Strategies

Mathematics has been considered more than a study of simple computation. Young children solved problems with various strategies in non-school settings (Carpenter et al., 1988). However, some of those same children had difficulty with school tasks concerning computation and meaningful problem solving. They were not sure if the answers even made sense. The main difference between a non-school setting and a school setting was that the school depended upon written symbols. If students were allowed to manipulate symbols only in a rote way, they had some difficulties (Hiebert, 1988). Children needed to connect symbols with understandings they had from experiences outside and inside of school. Connections were made only when they could: (a) interpret the symbols appropriately, (b) use well-understood strategies to manipulate the symbols, and (c) judge the reasonableness of the answer (Hiebert, 1988).

Research completed by Robert Balfanz (1988) considered how children used mathematical knowledge gained through experiences outside school. His argument was that the quality of elementary school mathematics was improved when teachers began with the knowledge that students had already developed. Another point he made was that children brought more knowledge to school today than children of 50 years ago, yet curricular materials have failed to take this into account. Their knowledge depended upon the complexity of mathematical experience in out-of-school activities rather a factor of environment, gender, or history (Balfanz, 1988).

Effects of Textbooks on Understanding

Textbooks have often been used to establish procedures for getting the correct answer rather than as instructional tools for developing multiple strategies (Balfanz, 1988). Balfanz based his study on the conjecture that primary textbooks lowered the quality of elementary education for some students if they were deprived of their local knowledge. His research reiterated that curriculum materials as well as instructional practice must be meaningful to the student and relate to applications in his/her daily life.

The concern of Balfanz (1988) is not an attack on textbooks or testing but to help the reader see that textbooks and tests must be used wisely as learning tools and not as the only sources of mathematical procedures. The most correct procedure was always the one that allowed the student to obtain the right answer. This was not always the procedure promoted in the text or prescribed by the test (Balfanz, 1988).

The concern about textbooks was also evident in the Stevenson et al. (1990) study comparing Japanese, Chinese, and American first and fifth graders. It was found that elementary textbooks in the United States obviated the need for children to attend closely to the word problem. Some elementary textbooks have taken one-step word problems and presented them many times by changing only the numbers. As a result, children quickly learned to focus on key words for determining the operation. Once they determined the operation to use in the word problem, they abandoned further efforts at analysis (Stigler et al., 1990). The NCTM <u>Standards</u> (1989) suggests decreasing attention on clue words for determining operation in order to increase a thought process.

Other Concerns with American Textbooks and Curriculum

The size of the American textbooks interfered with comprehensive coverage of first and fifth-grade curriculum in the Japanese, Chinese, and American study (Stevenson et al., 1990). Taiwan and Japan used smaller and shorter paper bound books that were completed during the year. The length of the American volumes often prevented teachers from completing the coursework or from following a set curriculum.

Taiwanese and Japanese textbooks had fewer illustrations and many were less explicit than American ones. For example, the Japanese textbooks did not show how to carry when adding two three-digit numbers, even though later problems clearly assumed this ability. Taiwanese and Japanese teachers were expected to elaborate the content of each lesson and supplement the content of the textbook by providing creative examples and interesting discussions as well as exercises for drill and practice (Stevenson et al., 1990).

In contrast, all the steps in the construction of a concept or skill were detailed in the American textbooks.

Textbooks did not require active participation in the development of the concepts, contrary to the Madeline Hunter (1980a) belief. American children simply followed the successive steps they had been taught (Stevenson, et al., 1990).

In the same study it was also noted that American textbooks suggested shallow mastery. American children were not asked to fully master concepts and apply them accurately. Concepts were often treated in a simplistic way. For example, in upper elementary textbooks probability concepts were introduced, but the ways in which probability theory could be applied were never discussed. In the Japanese curriculum, however, skills were immediately applied and developed (Stevenson et al., 1990).

Oral Discussion and Sufficient Feedback

Students were found to be their best teachers, if the classroom teacher used their experiences for lesson design and communication opportunities (Balfanz, 1988). However, the same research suggested that a total reliance on student self-development was counter-productive. A student may have been told that his approach led to an incorrect answer, but he/she was seldom told why this was the case. Therefore, the student ended mathematics class not knowing what he/she did wrong. The students who were given sufficient feedback on their inventions were more successful than those who had not been given feedback (Carpenter et al., 1988).

Carpenter et al. (1988) found that first graders, after only two days of specifically designed instruction, were able to form connections between addition and subtraction stories and associated number sentences. The key to effective instruction seemed to be that students were asked to use symbols as records of something they already knew (Hiebert, 1988). The best way to accomplish this task was to provide oral discussion and activities with concrete materials, story situations, or other things children understood. The oral component was extremely important (Balfanz, 1988; Carpenter et al., 1988; Hiebert, 1988). Written symbols were introduced after the ideas were clear. In other words, young children developed meaning for written symbols by first working with real quantities in real situations. Later, they used numerals and operation signs to stand for the quantities and actions on quantities (Hiebert, 1988).

Importance of an Integrated Mathematics Curriculum with Supplementary Activities and Manipulatives

The literature strongly supports the use of appropriate manipulatives for the beginning phases of learning new concepts (Perry & Grossnickle, 1987). Perry and Grossnickle

(1987) strongly supported the appropriate use of manipulatives for the beginning phases of learning new concepts. It was noted by the researcher, however, they based their recommendation solely on six previous studies without reference to internal validity. Their actual study on 75 kindergarten and first grade classrooms was based on what percentage of teachers actually used manipulatives, and which manipulatives were used. The study was not based on achievement.

Researchers have found that students having instruction in which manipulative materials were used scored significantly higher on achievement tests than students who had instruction in which manipulatives were not used (Suydam & Higgins, 1977). Both concrete and pictorial treatments were found to be superior to symbolic treatments in effecting achievement. Their studies also indicated that manipulative materials or visual aids was an advantage in problem solving, addition, subtraction, multiplication, and fractions. The use of manipulatives was of some importance for all grade/age levels in the elementary school. Additional materials in the lessons appeared to be effective with children at all achievement levels, ability levels, and socio-economic levels (Suydam & Higgins, 1977).

Observations in the Japanese, Chinese, and American study (Stevenson et al., 1990) revealed a difference in the use of manipulatives. Taipei and Sendai teachers used manipulatives more frequently than American teachers. Every classroom in Taipei and Sendai was equipped with a mathematics "set" of colorful, interesting materials which were used extensively in illustrating and representing mathematics. American children who were deprived of frequent concrete experiences with mathematics operations and concepts were less able to solve problems involving the application of mathematics outside the classroom. Additionally, they were less able to express a clear understanding of the significance of the equal sign (Stevenson et al., 1990).

Importance of Estimation and Mental Mathematics

Children need guidance in developing the ability to solve new problems (Carpenter, Hiebert, & Moser, 1983). Carpenter and his colleagues showed that three preliminary characteristics were essential and have received too little attention in traditional mathematics classes. The characteristics needing more attention include estimation, mental solutions, and the association of language with an operation used in connection with concrete objects. Children need to reason without the aid of paper and pencil at each grade level if they are to solve problems in the real world (Carpenter et al., 1983; NCTM, 1989).

Part of the problem-solving process was checking the reasonableness of the solutions. If the child connected the written symbols with the quantity, he/she probably would be able to come up with an estimate to the solution. Children showed little awareness of answers that make sense in the Carpenter et al. study. There was an absence of connection at this level. Consequently, children did not have the ability to check whether an answer was reasonable (Carpenter et al., 1983). Estimation was a useful instructional approach that helped to make a connection between the symbol and the reasonableness of it. Estimation was another set of rules that could not be applied by rote. Estimating solutions and judging whether an answer was reasonable depended upon knowing what the symbols meant (Carpenter et al., 1983).

Importance of Classification, Patterning, and Extension of Counting

Carol Thorton was convinced of the value of counting on, counting back, auditory patterning for two or three counts, and visual patterning based on the 10-frame. These four counting skills were hidden prerequisites for number-fact learning in addition and subtraction (Thorton, 1989). The skills were integrated in an ongoing program with activities during, or distinct from, regular mathematics time.

Counting backwards was very useful before children learned to subtract. Some children counted backwards from 10 but not from 9, 13, or 16. A good method for counting backwards was to ask the child to count as he/she broke off a unifix cube from a train of cubes (e.g., a 13-inch train) (Baratta-Lorton, 1976).

Auditory patterning was useful for young children because the activities helped them internalize short counts and made finger counting unnecessary.

A good example of visual patterning was Lorton's number station with two-sided beans. The two-sided beans proved to be an effective activity for concept development and reinforcement because of the motivational incentive.

Frequent oral explanations by the learner was a better testing device than computation tests with paper and pencil. Children took responsibility for their own progress in learning patterns.

Baroody (1987) noticed in his research that the difficulties with basic numeration skills were traced to a lack of understanding about position, or place, for multi-digit numerals. Mastery of the powerful but abstract place-value concept was only achieved gradually. Children rather quickly and mechanically learned names, but needed a deeper understanding when it came to zero acting as a place holder. They needed to realize that there is a repeating pattern to the system. It was found more helpful to teach this difficult concept by building on children's informal mathematical knowledge (Baroody, 1987).

Importance of Teaching on a Three-Level Sequence Using Concrete Materials

Marilyn Suydam (1987) noted in her research that children frequently demonstrated a poor understanding of place value in the base-ten system. Knowledge of numerals needed to be coordinated with conceptual understanding of the numbers. She found that children could grasp "fiveness" much more readily than "tenness." Five served as a base to aid them mentally. Their use of five as a key number and "counting on" helped Japanese children in understanding and recall (Suydam, 1987).

Beverly Baker (1977) noted a difference between first graders being taught lessons on place value. One group was taught on a three-level sequence; first on the inactive level, second on the iconic (pictures) level, and third on the symbolic level. Another group of first graders were taught a two-level sequence from iconic to symbolic. A third group had been taught at the symbolic level only. The mean scores on a place-value posttest taken by pupils in both the three-level sequence and the two-level sequence were significantly higher than those scores of the control group (Baker, 1977).

The Baker research, on the other hand, conducted a very tightly controlled study on 110 first graders. The researcher developed her own lessons on place value for the three levels of representation (i.e., inactive, iconic, and symbolic). First graders taught on either the two-level sequence or the three-level sequence scored significantly higher than those taught on one level. The investigator taught each subject individually, following scripts to control for examiner bias and to reduce teacher variability. A reliable posttest and a retention test were also individually administered. The ANCOVA adjusted for differences. Baker's significant results would appear very generalizable to other populations. She did make the recommendation to replicate the study using different mathematical content.

Jaynie L. Parham (1983) compared the use and non-use of manipulative materials on student achievement. The conclusion was a positive effect on student achievement. Her multiple regression analysis indicated a positive effect on student achievement tests at the 85th percentile, while non-users scored at about the 50th percentile. Her study was very meaningful due to the fact she calculated a total of 171 effect sizes over a 15-year period by using only dissertations of good quality with two, three, or four treatment groups. She used specific criteria for selection of quality dissertations (Parham, 1983).

Two other researchers also found significant differences in mathematical achievement of kindergartners and first graders who were taught on a three-level sequence in Mathematics Their Way (Slakey, 1984; Uecker, 1987). Margaret Slakey found that kindergartners and first graders who used MTW performed significantly higher in ordering, classifying, patterning, and place value than children taught with a traditional paper and pencil approach. Mean scores were much higher on the Comprehensive Test of Basic Skills, a norm-referenced standardized test (Slakey, 1984). Violations to internal validity may have been a problem in her study. The study appeared to have been biased because of the instrumentation. The oral interview questions were coordinated with the Mathematics Their Way assessments. The oral assessment did not appear reliable because of a bias in favor of MTW.

Milton Uecker (1987) found MTW first graders to perform significantly higher on both a written and oral assessment than their counterparts who were taught with A-Beka philosophy. A-Beka (1981) emphasizes the training of mental ability through the presentation of "concrete" facts. The scores were significant in favor of MTW in the problem-solving interviews. The MTW group also scored higher than the A-Beka group on the word problem portion of the Iowa Test of Basic Skills. Uecker (1987) concluded that oral problem solving has a positive relationship to the manipulative and three-instructional-level format of MTW. However, his significant results on kindergarteners and first graders possibly had a Type I error due to instrumentation. Instrumentation, a threat to internal validity, was violated because the test was not reliable. The assessment contained too few oral problems for accurate results. The small number of story problems also presented a serious violation. A sampling of a few students for interviews would not represent a reliable test. Also, the reader had no way of knowing the student knowledge level before the treatment. A t-test did not allow for individual differences or differences in children among buildings. As long as the researcher had not used an ANCOVA which adjusts for differences, the statistical significance may have been attributable to chance and not to Mathematics Their Way. Maturation was also another concern due to the fact that kindergarteners and first graders change drastically throughout the school year.

General Summary

The NCSM and the NCTM have searched for ways to effect curricular changes in mathematics. Their intent is to better prepare students for the 21st century. Their guidelines, implications of Piaget's theory, and ideas generated from Outcome Based Education are resulting in restructuring of mathematics programs. Recent research on elementary instruction also reflects the mathematics goals for the future.

It would seem reasonable that children in the elementary grades require some usage of manipulatives because they are in the Piagetian stage of concrete operational thought (Piaget, 1967; Wadsworth, 1989).

Evidence from research indicated that meaningful lessons taught with a purpose had a greater probability of increasing achievement (Baroody, 1987; Hunter, 1980b; Spady, 1991; Stevenson et al., 1990; Stigler, 1990; Thorton, 1989; Wittrock, 1986).

Making application with what students learn outside of school and making connections of symbols with understanding were also found to be important (Balfanz, 1988; Carpenter, et al., 1988; Hiebert, 1988).

Evidence from research indicated that lessons using manipulative materials had a greater probability of increasing achievement (Balfanz, 1988; Carpenter et al., 1983; Hiebert, 1988; Parham, 1983; Perry & Grossnickle, 1987; Stevenson et al., 1990; Suydam & Higgins, 1977; Thorton, 1989).

The oral component and sufficient feedback to students was equally important as using manipulatives (Balfanz, 1988; Carpenter et al., 1988; Hiebert, 1988; Suydam & Higgins, 1977). The primary effect of manipulative materials seemed to enhance more communication and a quality conversation concerning mathematics.

Evidence also indicated that estimation and mental mathematics needed to be incorporated into the primary curriculum (Carpenter et al., 1983).

The importance of classification, patterning, and extension of counting was emphasized by Baratta-Lorton (1976), Baroody (1987), and Thorton (1987).

Significance was found in methodologies that were taught on a three-level instructional (Baker, 1977; Parham, 1983).

Two specific mathematics programs using manipulatives and a three-level instructional sequence were significant. Kindergarteners and first graders showed significantly higher scores using Mathematics Their Way when their counterparts used paper and pencil "product" approaches (Slakey, 1984; Uecker, 1987).

The Japanese, Chinese, and American comparison revealed some noteworthy differences in methodology for teaching mathematics (Stevenson et al., 1990; Stigler et al., 1990). The description of their beliefs about children learning mathematics successfully resembles OBE philosophy because of the belief that "all children can learn" (Spady, 1991).

It would seem logical that mathematics instruction would be more meaningful in an Outcome Based Education structure because of the recent research on demonstrable outcomes in the OBE model (Spady, 1991), OBI model in the Glendale, Arizona, school district (Deever, 1991), and Outcomes-Driven Developmental Model in Johnson City, New York (Mamary, 1990). The premises of OBE very definitely have implications on mathematics in the classroom that will involve not only looking at new instructional techniques and programs with different methodologies, but looking at a new belief system.

Chapter 3

METHOD

Introduction and Purpose

The intent in this dissertation is to review the literature on how successfully early elementary children learn mathematics with the aid of manipulatives. The target will involve the comparative effectiveness of three methods of teaching mathematics to second graders.

According to Outcome Based Education (OBE) philosophy, "schools control the conditions" for children learning successfully (Spady, 1991). The researcher intended to find out differences in the three methodologies and implications for mathematics education in the elementary school.

The three treatments were designed to be representative of three different approaches to second grade mathematics instruction and were labeled as follows:

1. Treatment T for Scott Foresman (1990) textbook entitled *Exploring Mathematics*
2. Treatment M entitled *Mathematics Their Way*
3. Treatment C for University of Chicago School Mathematics Project (1991) entitled "Second Grade Everyday Mathematics"

Treatment T, a textbook curriculum, consisted of students using a formalized traditional approach with a heavy emphasis on paper and pencil computation. The book was designed specifically for the second grade. The manual describes the instruction as a concrete-pictorial-symbolic sequence. It would appear, however, that due to the use of counters as manipulatives in getting the right answer there is an abundance of work pages involving computation on the abstract level. The parental involvement consists of sending worksheets home for drill and practice. The textbook is very logically sequenced with additional practice worksheets for students who need remediation or enrichment. A problem solving and critical thinking sourcebook is another resource used in the classroom.

Treatment M was a developmental approach based on Piaget's theory with a heavy emphasis on the use of manipulatives and the process of learning mathematics in three phases (i.e., on the concept level, on the connecting level, and on the symbolic level). Manipulatives are used in order to understand the mathematical concepts through student/teacher interaction. Paper-and-pencil computation is minimal. It is not designed specifically for second grade. The manual is designed as a resource for all primary teachers. Treatment M teachers had agreed to collaborate and design their own problem-solving work packets in accordance with Mathematics Their Way by teaching on the symbolic level. The district allowed Treatment M teachers collaborative staff development time for designing their own problem solving practice sheets for the pilot program.

Treatment C was an integrated manipulative approach in which mathematics instruction is integrated into other curriculum areas. Special emphasis is placed on the language of mathematics as well as the use of applications in science, social studies, and the visual arts. It is specifically structured for second graders.

Instruction was activity centered with the intention of helping children with the transition between verbal, concrete, pictorial, and symbolic representations. The hands-on materials were used across the curriculum as well as in the home with parental involvement. Application type activities were sent to parents. Paper and pencil computation exercises used a somewhat different format than Treatment T as indicated by the samples in Appendix E. For example, in Treatment C, the second grader worked on the following drill and practice sheet for fact families.

Fill in the blanks with a plus or minus sign.
Do this in two different ways:

15 ___ 8 ___ 7
15 ___ 8 ___ 7

In Treatment T, the fact family work sheet looked like the following:

Subtract.

$$\begin{array}{r} 15 \\ -\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 15 \\ -\ 7 \\ \hline \end{array}$$

Treatment C was the only treatment using calculators to help learn the concepts. For example, in the above Treatment C exercise, second graders were told to answer without the calculator. They used the calculator to check their answers.

Research Questions

The quasi-experimental study attempted to answer the following questions with Treatment T as a traditional textbook approach, Treatment M as a manipulative approach, and Treatment C as a manipulative plus calculator approach:

1. Will mean posttest scores of children taught mathematics by Treatment T differ from children taught by Treatment M/Treatment C when controlling for pretest and I. Q. scores?
2. Will mean posttest scores of children taught mathematics by Treatment M differ from children taught by Treatment C when controlled for pretest and I.Q. scores?

3. Will mean posttest scores of children taught mathematics by Treatment T differ from children taught by Treatment M/Treatment C when controlled for pretest and age?
4. Will the posttest score means of children taught mathematics by Treatment M differ from those children taught by Treatment C when controlled for pretest and age?
5. Will mean posttest scores of children taught mathematics by Treatment T differ from children taught by Treatment M/Treatment C when controlling for pretest, age, and I.Q. scores?
6. Will mean posttest scores of children taught mathematics by Treatment M differ from children taught by Treatment C when controlled for pretest, age, and I.Q. scores?

Selection of Students

The population for the sample was in the Farmington, Minnesota, School District #192, located in the outer suburban ring of Minneapolis and St. Paul. The district included children from low, middle, and high socio-economic families. Each school in the district has approximately the same number of Chapter I students and low socioeconomic children in their buildings. For example, the Chapter I lead teacher reported in the annual evaluation that 49 second graders received services from Farmington Elementary School out of a total of 127; 40 Akin Road second graders received services out of a total of 123 (Swinehart, 1992).

All 10 sections of second graders were in one of the three treatment groups which included approximately 250 students. Students were assigned to similar academic groups in the five classrooms in the Farmington Elementary Building and five classrooms in the Akin Road Elementary building.

Second grade children were assigned according to a District #192 procedure. The scheduling procedure did not allow tracking or ability grouping. Teachers helped in the process by filling out cards on each student at the end of each school year as to whether they would fall into a low, middle, or high ability category. Low, middle, and high ability cards are arranged in three piles by the principal who selects the same number of students from each pile for a section. A few parent requests were granted before May 1 for the following school year; however, the ability range was taken into consideration so that each class had a mixture of remedial, average, and accelerated children. This procedure was used in June before the class lists were made available to the teachers and parents.

Due to those parent requests during the spring of each school year, random assignment was impossible. However, in accordance with Outcome Based Education philosophy and research by Goodlad and Oakes (1988) the Farmington School District complied with the idea that schools must provide all children with equal access to knowledge. The comparison studies of mixed groups found that, although all students experienced gains in language and mathematics, remedial students received the most gains (Goodlad & Oakes, 1988).

If there were discrepancies between sections or buildings, an Analysis of Covariance (ANCOVA) design was used to analyze the results. The ANCOVA has the property of statistically adjusting for initial differences between groups. The ANCOVA is not completely equivalent to random assignment of subjects, but it was the best alternative since the researcher could not select comparison groups that were matched to all relevant variables (Borg & Gall, 1989).

Most of the students had received two years of Mathematics Their Way in kindergarten and first grade. Students had spent much of their time using manipulatives on the concept and connective level for the previous two years. The exception would be a few children who had moved into the district during those two years and or had started during the 1991-1992 school year. They were assigned to sections by the same procedure as explained.

Selection of Classroom Teachers

The researcher chose classrooms only in the Farmington district to control variables among teachers as much as possible. All 10 teachers in the study have been trained in a six-day, 4-quarter-hour graduate credit workshop in the use of manipulatives in the classroom through the Center of Innovation in California.

All 10 teachers were trained in a four-day Madeline Hunter workshop in "Basic Elements of Instruction" through the district staff development program with district teacher/trainers. This procedure included observations and feedback to every second grade teacher by a staff development trainer and by the building principal.

Each second grade teacher had been trained in using cooperative learning in a heterogeneous group situation by the district staff development trainers. They had an equal number of evaluations in using cooperative learning by the building principal.

Outcome Based Education has become a part of the teacher repertoire. They have been involved in writing mathematics exit course outcomes for second graders and techniques in the classroom for mastering outcomes. The outcomes/assessment/report card for mathematics includes the number of problems that were tested and the percentage required for mastery. Each second grade parent received the checklist of outcomes and a letter from the researcher's office explaining the process for the school year. The outcomes that were written for the 1991-1992 school year are in Appendix G. Second grade teachers worked on the same outcomes but piloted different approaches to the outcomes.

Teachers from both buildings were allowed to volunteer for one of the three treatments in order to minimize the Hawthorn effect of the chosen approach. All teachers had the advantage of the "newness." Each teacher felt he/she chose the best approach in mastering second grade outcomes.

Experimental Design and Instrumentation

The researcher chose a pretest-posttest, control design with three treatment groups. The pretest acted as the control. Entry level knowledge of students was controlled by using pretest, I.Q., and age as covariates. The identical mathematics achievement pretest and posttest was given at the beginning of the first quarter of second grade and at the end of the third quarter.

The dependent variable, the applications portion of the California Diagnostic Mathematics Test (CDMT, 1989), has consistency of test results according to the test manual. The Kuder-Richardson formula 20 (KR-20) was applied to CDMT for evidence of reliability. The Kuder-Richardson, frequently used to measure internal consistency, has been shown to be a ratio of true score to true score plus error. Estimated KR-20 coefficients based on number-correct scoring of CDMT are found in Tables 4 through 12 in the California Diagnostic Mathematics Tests Technical Report (CDMT, 1989b).

The CDMT was chosen as the dependent variable for the pretest and posttest because it proposed to measure student application to story problems in mathematics which is what the researcher intended to assess for making comparisons. Therefore, it was a valid test.

Instrumentation in Relation to National Council of Teachers of Mathematics Standards

The message in the NCTM *Standards* is that knowing mathematics entails more than being skillful in performing mathematical procedures in isolation. The California Diagnostic Mathematics assessment questions satisfy the NCTM *Standards* and the recommendations made by the National Council of Supervisors of Mathematics with respect to alignment of the assessment instrument to the goals and topics specified in the curriculum. In order for the instrument to be aligned it must be in agreement with the outcomes, content, and instructional approaches of the curriculum (NCTM, 1989).

A standardized norm referenced test was chosen in conjunction with the Chapter I mandate. The applications portion of the California Diagnostic Test was in alignment with the second grade outcomes. One-third of the story problems focus on process rather than the correct answer. One of those questions, for example, is:

There were 17 apples in the basket. The children took 8 of them. How many are left?

$$\begin{array}{r} 8 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 17 \\ -\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 17 \\ +\ 8 \\ \hline \end{array} \qquad \begin{array}{r} 8 \\ -\ 8 \\ \hline \end{array}$$

The importance in the primary grade levels, as defined by district #192, is to demonstrate understanding of mathematical concepts. The process word problems measure the student's ability to set up a word problem for solution. In this study, the researcher wanted to measure application to problem solving. The dependent variable measures computational skills within the contexts of applications and concepts rather than skills in isolation.

Instrumentation as it Relates to Outcome Based Education

Additionally, this particular dependent variable supports the research on Outcome Based Education (Spady, 1991), the Outcome Based Education/Outcome Based Instruction

Model in Glendale, Arizona (Deever, 1991), and the Outcomes-Driven Developmental Model (ODDM) in Johnson City, New York (Mamary, 1990). Alignment of outcomes with the assessment is emphasized by all three proponents of Outcome Based Education (Deever, 1991; Mamary, 1990; Spady, 1991).

According to Outcome Based Education, a tight alignment needs to exist between curriculum techniques, outcomes, and assessment (Minnesota Department of Education, 1990b; Spady, 1991). One of the best ways to create alignment is to match the assessment with what is taught. "All children can learn" is an Outcome Based Education premise (Spady, 1991). The chosen dependent variable was a tool for all children including those with special needs. The assessment yielded reliable valid norm-referenced scores for children in the lower percentile because of a carefully controlled vocabulary as well as being based on current theory and practice. The print was larger for young test takers than most standardized tests. In some cases traditional standardized achievement tests are not very reliable for students falling below the 23rd percentile (CDMT, 1989a).

Outcome Based Education emphasizes teaching techniques that reflect life-like situations because of the statement, "What do we want the student to know, do, and be like upon graduation?" (Spady, 1991). The test was written by a professional team of experienced teachers and mathematics specialists that support the teaching process by selecting skills that can be transferred to applying strategies in life situations.

Practical Significance for Implications to Education

The researcher was interested in practical significance as well as statistical significance. The Delta was used to look at the practical significance of a statistic. The implications of practical significance are quite high. In this study, a Delta formula (Glass, McGaw, & Smith, 1981) was used to test the practical significance of the three treatments:

$$\text{Delta} = \frac{\bar{X}e - \bar{X}c}{Sc}$$

The experimental group means minus the control group means divided by the standard deviation of the control group yields the comparison. If the Delta figure is more than 0.4 for a particular treatment, cost and efficiency is fairly high (Glass et al., 1981).

The board members in the district were interested in any implications to mathematics education because of complaints from parents about the program and past mediocre scores.

Limitations of the Study

The study was limited to three different instructional approaches to teaching mathematics. Treatment T, Scott Foresman (1990) textbook, was very limited in using manipulatives while Treatment M, Mathematics Their Way, and Treatment C, University of Chicago Math Project (1991), placed a heavy emphasis on learning mathematics with manipulatives. Both Treatment M and Treatment C concentrated on three different levels of teaching new concepts with manipulatives and activities before reaching the symbolic level. Treatment C used calculators in the instruction because it was essential to that methodology.

These particular programs were chosen because of the availability of staff development in Minnesota for Mathematics Their Way (MTW) and University of Chicago School Mathematics Project (UCSMP, 1991). Other manipulative programs were available but the cost to send teachers to out-of-state staff development and training workshops was prohibitive.

The UCSMP was based on its usage in the high school. It is costly in the elementary grade levels because of the activity centered materials, calculators, consumable journals, and resource manuals for the teachers. It was limited to second graders in District #192 in Farmington, Minnesota. The Chicago Mathematics Project is only available in kindergarten, first, and second grade at the present time.

Teachers differed in years of experience. There was a range from 26 years of experience to 2 years of experience. Treatment T teachers had 23, 24, or 26 years of experience. Two Treatment M teachers had 4 years of experience. One Treatment M teacher had 16 years of experience. Two Treatment C teachers had 2 years of experience. One Treatment C teacher had 13 years of experience and one had 16 years of experience. Those who felt very strongly about some traditional methods volunteered for teaching with the basal approach using a textbook. Younger teachers who were less traditional volunteered to use MTW or UCSMP. The fact that teachers with the least experience in Treatment C may have introduced a bias in this study. (See Table 1.)

The study did not indicate the effect of teacher attitudes on students' mathematical achievement.

The research did not measure student attitude toward mathematics.

The research did not measure any interaction results between the treatment and individual teachers.

Data Analysis

In order to ensure that the treatment differences resulted in any gains, and not other factors, Analyses of Covariance (ANCOVA) were planned. The ANCOVAs compensated for academic differences, pretest, I. Q., and/or age among groups determining whether mean differences really existed or that observed differences were attributable to chance.

Table 1

Ten Participating Classrooms

Treatment	Years of Experience Farmington Elementary	Akin Road Elementary
T	23	
	24	
	26	
M		4
		4
		16
C	2	13
	2	16

It was expected that differences between the two schools or differences between sections of second graders could exist on pretest, age, and I.Q. Hence, the ANCOVAs adjusted for any differences. The adjustment effect compensated for the disadvantage. The posttest scores increased to a level that could predict on the basis of the correlation between the pretest and posttest (Borg & Gall, 1989).

The three independent variables were the teaching methodologies (i.e., Treatment T, as Scott Foresman [1990] Exploring Mathematics textbook, Treatment M, as Mathematics Their Way, and Treatment C, as University of Chicago School Mathematics Project [1991]). The null hypotheses were tested on the basis of a 0.05 level of significance.

Assumptions in Regard to Internal Validity

The researcher assumed that the 10 teachers in the study used their expertise from staff development training and their prior teaching experience for reliable results in each of the methodologies. The researcher gave second grade teachers a choice for the methodology. It was assumed that teachers who volunteered would aid in "staying on track," hence, giving support to that particular program. The intention was to avoid contamination in the study. It was assumed, therefore, that they would teach the program they volunteered to teach. Volunteers for a specific methodology were considered to eliminate some of the threats to internal validity mentioned in Educational Research (Borg & Gall, 1989).

The researcher administered the tests and completed the scoring in order to minimize differences in test administration. The same procedure was used in each classroom involving time, directions, and seating arrangements. The teacher remained in the classroom in order to monitor.

The researcher met with the teachers at the beginning of the school year in order to establish criteria for the quasi-experiment. A memo for the first meeting is located in Appendix H. Questionnaires, observation sheets, and monthly calendars were handed out for explanation.

All teachers were to comply with the school schedule for 40 minutes of daily mathematical instruction. All students were to receive the same schedule of 45 minutes in the computer laboratory for mathematics activities on the computers every six days.

The researcher observed and monitored the 10 classrooms for any discrepancies in order to control variables as well as met with teachers on a regular basis. This also assisted in assuring that the instruction was resulting in any difference and not outside variables.

The researcher monitored the classrooms during the quasi-experiment to observe that teachers were staying on task according to criteria. First, the researcher made every effort to control the seven threats to internal validity. Maturation is a concern with subjects in the early years. Second graders do mature at different rates during a school year. Second, the subjects ranged in age from seven years and six months to nine years and six months. The purpose of the study, however, was to see if there was a difference in achievement of this age group. The researcher eliminated any threat due to maturation by using age as a covariate in analysis.

The threat to instrumentation was minimized by orienting the teachers on procedures prior to the posttests. Additionally, the researcher administered pretests and posttests using the same procedures in all 10 sections. A memo is located in Appendix I. One alteration occurred whereby an emotionally behavior disordered student completed the test out of the classroom during the posttest.

Pretesting was a possible threat to the validity of the research because the posttest was identical to the pretest. Achievement tests sometimes familiarize the students with the type of questions they will deal with later and may artificially inflate the scores. However, in this case, the familiarity was intended as clearly focusing on the outcomes for second grade, as in Outcome Based Education philosophy (Spady, 1991).

The possibility of regression as a threat enters into the situation when a pretest and posttest are given. In this study, a reliable instrument was chosen. Regression occurs when the group being measured is selected on the basis of extreme scores, the group is measured with an unreliable instrument, and/or the intervals between testing are lengthy. All second graders in the district were selected as subjects in order to minimize regression to the mean. The instrument, the California Diagnostic Mathematics Test (CDMT, 1989), purports to provide norm-referenced information that is reliable for evaluating progress and for fulfilling state and federal requirements. Second, the instrument was in concert with OBE teaching methods because it addressed the needs of all students by reducing the frustration sometimes experienced with standardized tests. The dependent variable uses two modalities instead of one. Students not only hear the story problem as in other standardized tests, but also are able to see the words in front of them at the same time. The two modalities are more apt to help the child to be more successful on the assessment. Children need to feel successful in test taking according to Spady (1991) because of the premise that "success breeds success." Regression proposes a slight threat, however, because of the 27 weeks between testing occasions.

Selection did not propose a serious threat due to the composition of the group. The range of abilities indicate a mixed cross section of second graders. As measured by the Test of Cognitive Skills, abilities ranged between 64 and 141. A serious threat arises only when the composition of the group itself rather than the treatment produces the outcome. There is no reason to believe that was the case.

Mortality is an effect due to subjects dropping out of the treatment on a non-random basis. Ten students did not complete the posttest because their parents moved out of the district, but not due to experimental mortality. Their reason for leaving had nothing to do with the treatment. The small number of students did not pose a serious threat because of the large number of subjects, 140, who participated from start to finish.

History, one of the major threats to internal validity, was minimized by closely monitoring the classrooms, meeting with teachers, and checking on lesson plans. Computer lab lessons were kept on identical time lines. Time on mathematics sessions was consistent in all classrooms in both schools. All teachers had agreed to use Arithmetic Developed Daily three times each week which added four mixed-story problems during each mathematics session. All teachers had the same amount of teacher assistance time in helping students to master the concepts. Teachers kept a journal in order to assure adherance to time lines. The researcher met with teachers ahead of time for explanations in preparation for the study. The fact that teachers could select one of three methodologies allowed them to approach a mathematics treatment with a positive attitude. The steps would not totally eliminate the history threat but certainly reduced a serious threat to the internal validity of the conclusions.

Chapter 4

ANALYSIS OF THE DATA

Introduction and Restatement of Hypotheses

The investigator will restate the hypotheses for clarity in the study:

1. There is no statistically significant difference in the posttest score means on a mathematics applications assessment between a traditional textbook approach and other manipulative approaches when the pretest and I.Q. are controlled.
2. There is no statistically significant difference in the posttest score means on a mathematics applications assessment between Mathematics Their Way and University of Chicago Mathematics Project (1991) when the pretest and I.Q. are controlled.
3. There is no statistically significant difference in the posttest score means on a mathematics applications assessment between a traditional textbook approach and other manipulative approaches when the pretest, and age are controlled.
4. There is no statistically significant difference in the posttest score means on a mathematics application assessment between Mathematics Their

Way and University of Chicago Mathematics Project (1991) when the pretest, and age are controlled.

5. There is no statistically significant difference in the posttest score means on a mathematics applications assessment between a traditional textbook approach and other manipulative approaches when the pretest, I.Q., and age are controlled.
6. There is no statistically significant difference in the posttest score means on a mathematics application assessment between Mathematics Their Way and University of Chicago School Mathematics Project (1991) when the pretest, age, and I.Q. are controlled.

Results

The data from the pretest, posttest, I.Q. test score, and age of subjects, shown in Appendix J, was entered on a VAX 4000 mainframe computer using the Statistical Analysis System, (SAS) Version 5.14.

In the first analysis of covariance, no statistically significant differences were observed. The analysis of covariance is shown in Table 2. The pretest was used as the covariate and the posttest as the dependent variable. Given that pretest differences were controlled for between the groups, there was not enough evidence (F [1, 236] = 1, p = .3175) to conclude that Treatment T differed from others in the posttest score.

Table 2

Planned Comparison of Treatment T vs. Others Using Analysis of Covariance with Pretest as Covariate[a]

	Degrees of Freedom	Mean Squares[b]	F-Ratio	PR) F
Between	1	23.6820	1.00	.3175
Within	236	23.5975		
Total	237			

[a] California Mathematics Diagnostic Test, Level B

[b] Planned comparisons based on mean square error

Given that pretest differences were controlled between the groups, there was not enough evidence (F [1, 236], p = .0625) to conclude that Treatment M differed from Treatment C on the posttest score. The figures are provided in Table 3.

A second ANCOVA was used with the pretest and I.Q. score as covariates and the posttest as the dependent variable. Given that pretest and I.Q. differences were controlled between the groups, there was not enough evidence

(F [1, 235] = 1.11, p = .2930) to conclude that Treatment T differed from others in the posttest score. The figures are provided on in Table 4.

Table 3

Planned Comparison of Treatment M vs. Treatment C Analysis of Covariance with Pretest as Covariate[a]

	Degrees of Freedom	Mean Squares[b]	F-Ratio	PR) F
Between	1	82.6884	3.50	.0625
Within	236	23.5975		
Total	237			

[a] California Mathematics Diagnostic Test, Level B

[b] Planned comparisons based on mean square error

Given that pretest and I.Q. differences were controlled between the groups, there was not enough evidence (F [1, 235] = 1.71, p = .1927) to conclude that Treatment M differed from Treatment C in the posttest score. The figures are provided in Table 5.

Table 4

Planned Comparison of Treatment T vs. Others - Analysis of Covariance with Pretest and I.Q. as Covariate[a]

	Degrees of Freedom	Mean Squares[b]	F-Ratio	PR) F
Between	1	23.9172	1.11	.2930
Within	235	21.5302		
Total	236			

[a] California Mathematics Diagnostic Test, Level B

[b] Planned comparisons based on mean square error

Table 5

Planned Comparison of Treatment M vs. Treatment C - Analysis of Covariance with Pretest and I.Q. as Covariate[a]

	Degrees of Freedom	Mean Squares[b]	F-Ratio	PR) F
Between	1	36.7469	1.71	.1927
Within	235	21.5302		
Total	236			

[a] California Mathematics Diagnostic Test, Level B

[b] Planned comparisons based on mean square error

A third ANCOVA was used with the pretest and age as covariates and the posttest as the dependent variable. The ANCOVA is shown in Table 6. Given that pretest and age differences were controlled for between the groups, there was not enough evidence (F [1, 235] = 15, p = .2850) to conclude that Treatment T differed from others in the posttest score.

Table 6

<u>Planned Comparison of Treatment T with Others - Analysis of Covariance with Pretest and Age as Covariates</u>[a]

	Degrees of Freedom	Mean Squares[b]	F-Ratio	PR) F
Between	1	26.5694	1.15	.2850
Within	235	23.1382		
Total	236			

[a] California Mathematics Diagnostic Test, Level B

[b] Test of Cognitive Skills

Given that pretest and age differences were controlled for between the groups, there was not enough evidence (F (1, 235) = 3.56, p = .0603) to conclude that Treatment M differed from Treatment C in the posttest score. The tables are provided in Table 7.

A fourth ANCOVA was used with the pretest, I.Q., and age as covariates and posttest as dependent variable. Given that pretest, I.Q. and age differences were controlled for between the groups, there was not enough evidence (F [1, 234] = 1.20, p = .2751) to conclude that Treatment T differed from others in the posttest score. Figures are indicated in Table 8.

Table 7

Planned Comparison of Treatment M vs. Treatment C - Analysis of Covariance with Pretest and Age as Covariates[a]

	Degrees of Freedom	Mean Squares[b]	F-Ratio	PR) F
Between	1	82.4248	3.56	.0603
Within	235	23.1382		
Total	236			

[a] California Mathematics Diagnostic Test, Level B

[b] Test of Cognitive Skills

Given that pretest, I.Q., and age differences were controlled for between the groups, there was not enough evidence (F [1, 234] = 1.82, p = .1785) to conclude that Treatment M differed from Treatment C in the posttest score. Figures are provided in Table 9.

Table 8

Planned Comparison of Treatment T vs. Others - Analysis of Covariance with Pretest, I.Q., and Age as Covariates[a]

	Degrees of Freedom	Mean Squares[b]	F-Ratio	PR) F
Between	1	25.6381	1.20	.2751
Within	234	21.4240		
Total	235			

[a] California Mathematics Diagnostic Test, Level B

[b] Planned comparisons based on mean square error

Table 9

Planned Comparison of Treatment T vs. Treatment C - Analysis of Covariance with Pretest, I.Q., and Age as Covariates[a]

	Degrees of Freedom	Mean Squares[b]	F-Ratio	PR) F
Between	1	39.0060	1.82	.1785
Within	234	21.4240		
Total	235			

[a] California Mathematics Diagnostic Test, Level B

[b] Planned comparisons based on mean square error

Additional treatment and overall results are listed in Appendix K.

Even though the ANCOVA indicated no statistically significant results, the Delta formula (Glass, McGaw, & Smith, 1981) indicated that Treatment M had a larger effect size than Treatment M. The researcher used the Delta formula (i.e., $\bar{X}e - \bar{X}c$ / Sc, or the mean of the experimental group minus the mean of the control group divided by the standard deviation of the control group). Table 10 shows the calculations indicating the larger effect size of Treatment M where students were taught <u>Mathematics Their Way</u> along with problem solving worksheets designed by the second grade teachers. The means of the gain scores are indicated in the Analysis of Covariance Cell Values in Appendix K. Treatment T and Treatment C fell below the critical 0.4 mark indicating small effect sizes.

Delta is used to look at the practical significance of a statistic. However, the researcher would caution the reader that a Delta did not take into consideration any preexisting differences, nor did it control for age or I.Q. The Delta simply indicated that students in Treatment M made more gains than students in Treatment C or T. This can be shown by using .41 from Table 10 and changing it to a Z score which corresponds to the 66th percentile according to a statistic conversion chart (Ferguson & Takane (1989).

Using the data on a normal curve, the researcher concluded that Treatment M students made a 16% gain (60 - 50 = 16) that students in the other treatments did not make. This did not meet the conventional level of significance nor did it control for the pretest as was possible with an ANCOVA.

Table 10

Calculations Indicating the Practical Significance of Treatment M

Practical significance of Treatment M

$$\text{Delta} = \frac{\bar{X}e - \bar{X}c}{Sc} \qquad \frac{12.653 - 9.8}{6.891} = .414$$

No practical significance of Treatment C

$$\text{Delta} = \frac{\bar{X}e - \bar{X}c}{Sc} \qquad \frac{12.653 - 11.516}{6.714} = .1693$$

No practical significance of Treatment T

$$\text{Delta} = \frac{\bar{X}e - \bar{X}c}{Sc} \qquad \frac{11.516 - 9.8}{6.891} = .2490$$

Summary and Discussion

The statistical analysis, using the California Diagnostic Mathematics Test (1989) as the evaluative instrument, did support the null hypothesis that there is no difference in the effect on achievement gain score means when using a traditional textbook, Treatment T versus the others, that is, Treatment M and Treatment T instruction.

Using the same statistical analysis, the CDMT instrument also supported the null hypothesis that there is no difference on gain score means when using Mathematics Their Way, Treatment M, versus University of Chicago Mathematics Project (1991), Treatment C instruction.

All three treatments had covered the content of the evaluative instrument, that is, story problems, measurement, counting money, telling time, and graphing. All test items were applications to mathematics. All five topics had been presented in the three treatments; however, they were not identical in format. Treatment T children had been accustomed to a structured workbook format with pictorial representation. Treatment M children had additional exposure from the previous two years to concrete manipulatives for counting money and for working problems. Additionally, they were exposed to problem solving practice sheets designed by Treatment M teachers to compliment Mathematics Their Way. Treatment C children used calculators as tools in learning to count money and to solve problems.

All treatment groups worked toward the same outcomes and used the same mathematics report card with outcomes listed and percentages of problems expected to master during the second grade year. All second graders were given the oral assessments at the end of first, second, and third quarter on the same list of outcomes found in Appendix D.

Chapter 5

SUMMARY, CONCLUSIONS, DISCUSSION, AND RECOMMENDATIONS

Restatement of the Problem

The researcher saw a need for restructuring the K-12 mathematics program due to the compliance with the NCTM Standards (1989). Results from large cross-national studies of elementary school children should not focus entirely on improving the performance of high school students (Stevenson et al., 1990; Stigler et al., 1990). Mathematics problems arise earlier. Improving mathematics performance in the United States is an important goal of the National Research Council (1989), National Council of Teachers of Mathematics (1989), National Council of Supervisors of Mathematics (1988), as well as the Minnesota Department of Education (1991), and educators in our schools. Improving secondary education is part of a major goal, but concentrating remedial efforts on secondary schools may come too late in the academic careers of students to be effective. Therefore, the primary grade levels were of particular interest.

Piagetian philosophy is embraced in the researcher's district providing developmentally appropriate curricula in the kindergarten and first grade levels. Second-grade teachers have received staff development training in the use of manipulatives but were particularly frustrated with changes recommended by the National Council of Teachers of Mathematics. Problem solving and multiple strategies for getting the right answers are stressed and yet students cannot problem solve and perform on standardized tests without knowing mathematical facts. Piagetian philosophy recommends understanding mathematical concepts before memorizing facts. Using manipulatives has brought many complaints by third grade teachers because of lower pretest scores at the beginning of third grade. The concern is what program to use in the second grade that will "bridge" the developmentally appropriate K-1 curricula with the textbook approach in intermediate elementary grade levels and high school.

The purpose of this investigation was to determine the effect of three different methodologies on mathematics application achievement of second graders. Two different manipulative approaches were compared to a traditional delivery using a textbook. Story problem/application achievement scores were compared to see if one of the new manipulative approaches would better meet the needs of students. Any significant differences would have implications for the adoption of a particular elementary mathematics program in compliance with the NCTM Standards.

Summary

Ten classes using three different methodologies were compared at two elementary sites. The classes consisted of second graders in the Farmington School District in the Akin Road Elementary and Farmington Elementary buildings. The second graders were exposed to one of three methodologies (i.e., Treatment T, Treatment M, or Treatment C). Outcomes for all second graders were the same and are found in the appendices. Units covered in mathematics were the same in all methodologies (i.e., counting, story problems with missing numbers, place value, money, time, measurement, and graphing).

The methods and techniques used by 10 second grade teachers were different: Treatment T, traditional textbook by Scott Foresman (1990), Treatment M, Mathematics Their Way with additional worksheets that were designed by three teachers using this methodology, or Treatment C, University of Chicago School Mathematics Project (UCSMP) (1991) with calculators. The UCSMP is in its early infancy having been written only for kindergarten, first, and second grade at the time of this study. This is another reason why the study was completed on second graders rather than upper grade levels.

The study focused on whether or not second graders became more effective problem solvers by memorizing number combinations through practice in written exercises or by interacting with concrete materials, grasping the meaning of concepts, and only then applying them. The former methodology was a descriptive feature in Treatment T. The latter methodology described Treatment M and Treatment C. Treatment C involved three additional components (i.e., parental involvement through "home links," journals with problem solving activities, and learning to use a calculator). The calculators were used as tools for drill and practice in learning addition and subtraction facts as well as checking multi-digit problems.

A quasi-experimental design with nonequivalent control-group management was conducted in the research. All three groups received a treatment and were administered a pretest and posttest. The results for the study were obtained by an evaluative standardized instrument, the California Diagnostic Mathematics Test (1989), level B, which was written for grade levels between 2.6 and 3.9. The instrument was selected because of its alignment with second grade mathematics outcomes and NCTM standards.

Conclusions

The data reveal no statistical differences in problem solving achievement in any of the three methodologies in the second grade classrooms.

Treatment M and Treatment C indicated higher gain scores than Treatment T. However, the gains would be attributed to chance because of the pretest, and I.Q. controls. The same was true when pretest and age were controlled.

Overall, there were only nine students who made no gains from the pretest. One student in Treatment T made a negative score in comparing pretest and posttest. Generally speaking, almost all students made good gains in achievement in all methodologies. Results are in the Appendix L. The "ceiling" effect may be attributed to the fact that all teachers taught to the same outcomes and all teachers supported the researcher by following the criteria set up at the beginning of the year. Those outcomes were orally assessed at the end of each quarter. The results support OBE philosophy in that outcomes may be reached using multiple resources depending on individual students (Spady, 1990). Each teacher used different resources depending upon the treatment.

In regard to the posttest, there were no "big" differences in means between programs. Seventy-five percent of all students scored above 80% in all three programs. The "ceiling" effect of all treatments tends to mask any big differences. The increased structure and direction in the experimental treatment had a positive effect on all students. In the researcher's study, the positive effects were noticed in all treatments rather than in only one specific treatment. The results seem to suggest that the teacher may have more to do with achievement gain than specific curricula.

The positive effect was also noticed in the Chapter I NCE mathematics gains made by second grade students as compared to gains made by other students in other grade levels and other subject areas (Swinehart, 1992). Table 11 shows evidence of mathematics gains.

Extreme scores at both ends were observed on the pretest with a mode of 23. Children on the lower end made the most gains with 25 students scoring at 79% or above on the pretest. The difficulty of the test might seem questionable; however, the evaluation standards emphasize that assessment and program evaluation should assess students on what they know and think about mathematics. The emphasis is "decreasing attention on what they do not know" (NCTM, 1989, p. 191).

There were no statistically significant differences between treatments when I.Q. was controlled even though Treatment T had all students with an I.Q. above 80; Treatments M and C had some students with I.Q.s in the 60 and 70 range. Treatment M classrooms had four students below 80 and Treatment C had five classrooms below 80.

Table 11

Farmington Elementary Schools Chapter I/AOM NCE Gains (June 1992)[a]

Grade	Number of Students	Math Total
5	26	8.7
4	18	8
3	33	11.9
2	31	17.8

[a] NCE gains according to California Diagnostic Mathematics assessment results from spring to spring testing. These gains are from 31 Chapter I/AOM students only.

Conclusions Regarding Manipulatives

Since there were no statistically significant differences between Treatment T and the others, the researcher concluded that the use of manipulatives did not necessarily guarantee meaningful learning. Manipulatives were effective, probably because children made connections between experience and existing knowledge as in the Balfanz (1988) research, or they required reflection on the part of the students. The particular medium (objects, pictures of objects, or video displays of objects used to a great extent in Treatment M and Treatment C) was probably less important than the fact that the experience was meaningful. This would support previous literature (Hunter, 1980b; Spady, 1991; Stevenson et al., 1990; Thorton, 1978; Wittrock, 1986). Gains shown in all treatments indicate that manipulatives are an additional tool for learning mathematics (Carpenter et al., 1988; Hiebert, 1988; Hunter, 1980a; Suydam & Higgins, 1977).

Discussion and Educational Implications

Even though this study indicated no statistically significant differences in the three methods of mathematical instruction, the large effect size of Treatment M occurred by using the Delta formula (Glass et al., 1981).

Getting an effect size of .41 would seem to indicate that Mathematics Their Way was meaningful. However, the effect size was calculated without the control of preexisting differences as in the ANCOVA. The favorable gain, though not statistically significant, has future implications for second grade education.

The researcher observed and documented, as well as evaluated, all second grade teachers during the treatments. The researcher noted during observations that second graders in Treatment M began their lessons as in Piagetian classrooms with manipulatives. As they progressed and matured with logical thought processes, they completed more abstract work sheets. Teachers were able to monitor and adjust (Hunter, 1992) by selecting a particular work sheet at the "right time" (Wadsworth, 1989) in accordance with Piagetian theory (1952). The intent of manipulatives in the classroom samples was to develop an understanding rather than to learn only rote facts. The dependent variable assessed computational skills at the comprehension and application levels. The educational methods used in Treatment M and Treatment C were consistent with how children learn through experience (Piaget, 1971). Students from both Treatment M and Treatment C were exposed to more active participation.

Treatment T teachers followed the format in their "step by step" manual, using pages of the workbook in the order given. Treatment M and Treatment C teachers were more inclined to watch for "spontaneous interests" (Wadsworth, 1989) and plan for the next lesson accordingly.

As in Outcome Based Education (Spady, 1991), all classrooms ascribed to a heterogeneous non-grouping model. Cooperative learning groups, characteristic of OBE (Deever, 1992; Mamary, 1990; Spady, 1991) were particularly noticeable in Treatment M and Treatment C.

The NCTM goals recommended meaningful instruction for the development of reasoning and understanding on a higher cognitive level (NCTM, 1989). Purposeful and relevant instruction were cited in the review of the literature as essential components for successful problem solving (Carpenter et al., 1988; Hunter, 1980b; Spady, 1991; Stevenson et al., 1990; Stigler et al., 1990; Thorton, 1978). Some meaningful instruction allowing for understanding and reasoning skills was demonstrated in all treatments during the researcher's observations. This may have attributed to the non-existent statistical significance. All teachers were very conscientious about teaching to the outcomes, characteristic of OBE (Spady, 1991).

Hiebert (1988) and Balfanz (1988) explained in their literature that children needed to connect the symbol with an experience in a "non-school setting" or "out of school activity." Teachers in all treatments afforded some activities honoring these requests. All teachers, having been trained in Madeline Hunter (1980b) techniques, used anticipatory sets that tied the lesson to some purpose relating to student experience.

Hiebert's (1982) study involved an oral interview with 47 first graders on their modeling behavior and solution processes with manipulatives. Even though the study was limited to children solving only six problems, it revealed some important data for teachers on appropriate story problem instruction.

Treatment M teachers did supplement the lessons with very creative examples, interesting discussion, and relevant practice exercises as did Japanese and Chinese teachers in the Stevenson et al. (1990) study. Treatment M lessons were more stimulating and motivating during the researcher's observations which might explain the practical significance of this study. Teachers could also monitor and adjust (Hunter, 1992) more easily because they did not have the step-by-step procedures in front of them as did the

Treatment M and Treatment C teachers. They could more easily allow for individual differences by letting some children continue with the concrete level and connecting levels, but challenging others with higher level problem solving.

The generative theory of learning (Wittrock, 1986) maintained that learners needed to generate meaning during a lesson. Other researchers such as Hiebert (1988), Balfanz (1988), and Carpenter et al. (1988) suggested that children need a purpose or sense of value in learning mathematical concepts. Likewise, the NCTM Standards (1989) alluded to the same goals as well as requested opportunities for "confidence" in being able to perform operations. The Minnesota Department of Education (1990c) also confirmed a goal involving "confidence" in mathematics.

Balfanz (1988) noted in his investigation that curriculum materials as well as instructional practice had to be meaningful to the student. His Chicago study of 40 second graders was somewhat limited for making generalizations because the sample was taken in a private school.

In a much more comprehensive cross-national study, Stevenson et al. (1990) presented the problem of textbooks.

The NCTM K-4 Standards (1989) suggest that textbooks or resources tie mathematics to communication, reasoning, connections, patterns, and relationships, as well as problem solving, estimation, geometry, measurement, statistics, and fractions. It would seem that creative teaching with an array of manipulatives might take the place of the more expensive consumable workbooks used in Treatment T or Treatment C. Treatment T teachers were the most experienced teachers having used a textbook approach for 23, 24, or 25 years. Two Treatment C teachers had 2 years of experience, and the others had 13 or 16 years. Treatment M teachers had the least experience (i.e., two with 4 years and one with 16 years).

Another factor may have made a difference in this study. It was noted by the researcher during several observations, that one of the Treatment M classes had three noticeably immature children plus one disruptive Educationally Behavior Disordered (EBD) student in the classroom. If anything, these factors would have contributed to fewer gains because these students did not stay on task without being closely monitored by the teacher during both the pretest and posttest. During the posttest, the disruptive student needed to leave the room with the EBD teacher and complete the test upon the return of normal behavior. The other three cried during the pretest but managed to maintain composure during the posttest. Treatment M teachers were the least experienced and seemed to have more problem students in their classrooms.

There is still another consideration relative to this study. It was suggested by the NCTM Standards (1989) that textbooks decrease attention on clue words in order to increase the thought process. Stigler et al. (1990) also alluded to the problem in comparing textbooks. Treatment M teachers were able to avoid at least some of the repetitious word problems associated with clue words. They relied totally on teaching to the outcomes rather than coverage of the entire second grade consumable book as in Treatment T or Treatment C.

Treatment M and Treatment C teachers relied more on active participation during concept teaching associated with the "connections" requirement of the NCTM Standards (1989) and used the Madeline Hunter (1992) model of overt and covert active participation. Treatment M and Treatment C children were allowed to use "non-school strategies" for connecting experience with symbols as in Hiebert's (1988) work. The concept of making "connections" is one of the goals of the NCTM Standards (1989). Treatment M and teachers allowed time for making "connections" during cooperative learning sessions at least twice a week. Treatment C teachers used cooperative groups once or twice a week. Treatment T teachers used cooperative groups less often according to the observations and questionnaire in the Appendix. Treatment M and Treatment C children were exposed to more cooperative learning involving connections.

Communication was probably enhanced in both Treatment M classrooms and Treatment C classrooms. It appeared that students were given more feedback during their manipulative lessons and cooperative learning sessions in both Treatment M and Treatment C classrooms. The conclusions in the Carpenter et al. study (1988) found that students were significantly more successful if given feedback. In their study, first graders were extremely successful in solving simple addition and subtraction word problems. The oral component also enhanced learning according to Hiebert (1988) and Balfanz (1988).

The Carpenter et al. (1983) analysis suggested when symbolism was introduced prematurely, number concepts were not enhanced. Even with less drill and rote learning of mathematical facts, Treatment M children still outperformed other treatments. Therefore, it would seem logical and reasonable to delay symbolic addition and subtraction in kindergarten and first grade, as well as part of second grade. The mastery of addition and subtraction facts probably came more easily when a child had performed at an understanding level as evidenced by the similar results of all treatments. The majority of the second graders in this study had Mathematics Their Way in kindergarten and first grade. This may have contributed to similar results of the three treatments.

Treatment M and Treatment C classrooms both met the requirements described in the Stevenson et al. (1990) research comparing available resources to supplement the program. Taipei and Sendai students scored significantly higher where teachers used "sets" of manipulatives extensively.

The NCTM Standards (1989) and Carpenter et al. (1983) stressed the importance of estimation and mental mathematics. All treatments in this study were exposed to mental mathematics problems three days a week according to their agreement and schedule. Treatment M and Treatment C teachers used some clever ways to incorporate estimation into student lessons. Treatment M teachers involved parents in estimating activities during open house. Parent awareness places a "value" on mathematics, thereby staying in concert with the NCTM Standards (1989) and Minnesota Department of Education (1991).

During observations, the researcher noticed more visual and auditory patterning activities in the Treatment M classrooms. For example, at the beginning of every lesson, Treatment M teachers had counting activities using straws or popsicle sticks in order to explain place value. Children realized that "tenness" involved moving nine items to a new container representing the 10's place. Their methodology was in accordance with Baratta-Lorton (1976), Baroody (1987), and Thorton (1978).

The Thorton (1978) research indicated that second and fourth graders performed significantly better on addition problems by using supplementary aids beyond textbook curricula than others who used only the textbook. The sample, however, was very small. Random assignment was made to 25 second graders in the experimental group and to 22 in the traditional group. Likewise, the fourth graders had 23 in the experimental group and 20 in the traditional group. A Type I error was very possible due to the small sample size in the study and due to using an ANOVA. In other words, chance instead of the treatment may have attributed to the significance. Also, the ANOVA did not adjust for preexisting differences.

As observed by the researcher, Treatment M and Treatment C children learned place value using three steps: (a) instruction on the concept level, (b) instruction on the connecting level, and (c) instruction on the symbolic level (Baratta-Lorton, 1977). A similar procedure was used in UCSMP (1991) with only the terminology being different. Treatment T children followed the textbook with a pictorial and symbolic sequence. Treatment M children spent more lessons on manipulative activities involving place value on the three levels. For example, they experienced counting lessons using "base five" instead of "base ten" (Baratta-Lorton, 1977; Suydam (1987). After counting out five singular beads on the right side of the divided board, Treatment M children automatically placed them in a paper cup on the left side of the board to represent the 5's side. This procedure was consistent with Suydam's (1987) analysis on children grasping "fiveness" much easier than "tenness." The culminating lesson in Treatment M classrooms was using the same beads but with our base ten instead of base five. This exercise was used for both addition and subtraction problems before students were given double digit worksheets.

Treatment M teachers used Outcome Based Education philosophy (Spady, 1991) in their multiple strategies for different learning styles. The OBE was also reflected when they retaught information with different approaches. Treatment M teachers used several resources for a blended approach with the addition of some appropriate worksheets. The manual did not have a scope and sequence for second grade. It was used as a resource book for all primary grade levels. Mathematics Their Way is written in such a way as to provide suggestions for concept and connecting level teaching as described earlier. The generalized format was intended for kindergarten through third grade but makes no distinction between age or grade levels. The teacher is the decision maker on what to teach and when to teach depending upon when the primary child is ready for abstract symbols (Piaget, 1952). This approach is in alignment with OBE that when a student reaches one outcome, he/she advances to a more difficult one (Spady, 1991).

On the other hand, it is possible the selection of practice sheets used by the second grade teachers contributed to the lack of statistical significance.

Recommendations for Teachers and Administrators

Manipulative materials, when used creatively in the classroom with an outcome in mind, contribute to gain scores. Outcomes place a clear focus on what has to be taught. Manipulatives are a tool, however, and increase the effectiveness of instruction when used meaningfully. Manipulatives should not be used in a random, unorganized fashion. Children should not learn merely to manipulate objects. They are a means to attain important goals in mathematics instruction.

Creativity of the lessons generated by multiple resources, as opposed to a specifically set curriculum, is the key to improved mathematics achievement in the primary classroom. The intent of the creative lessons is to make mathematics a potentially exciting subject as in Chinese and Japanese classrooms (Stevenson et al., 1990). American teachers need to elaborate the content of each lesson by supplementing the content with more creative examples and more interesting discussion (Stevenson et al., 1990). This approach will help primary children to "value" mathematics (Minnesota Department of Education, 1990a; NCTM, 1989).

Implications to Curriculum Developers

Evidence suggests that if school districts are on a limited budget, a supply of manipulatives and multiple resources for the second grade teachers may be more important than the purchase of consumable workbooks. The focus needs to be on what is available for the teacher to teach effectively. This study is in concert with the research from the cross-national study (Stevenson et al., 1990; Stigler et al., 1990).

Second, it is essential to allocate more of the budget for training teachers through staff development in the use of manipulatives and multiple strategies. The additional materials made available in the classroom do very little for student achievement if teachers are not trained to use them. The effectiveness of Japanese and Chinese elementary schools can become more common in the United States. American educators need to look to Taiwanese and Japanese models because their teachers were allowed more staff development/ collaboration time with other teachers (Stevenson et al, 1990; Stigler et al., 1990). The liveliness and intensity described in Asian classrooms (Stevenson et al., 1990) will happen only with increased preparation/sharing time through out the school year.

The implications of this study confirm the belief written in the NCTM Standards (1989) that instruction and curriculum must be considered equally in judging the quality of a program.

Recommendations for Further Research

The effect size of Treatment M has some merit. The researcher would recommend a replication with more control and more power. More heterogenous mixing of teachers should be used in another study. Mixing inexperienced teachers in the three treatments would eliminate any bias.

The Stigler et al. (1990) and the Stevenson et al. (1990) studies did recognize the value of the "standardized curriculum" throughout all classrooms which seems somewhat contrary to the results found in this second-grade study and contrary to OBE philosophy (1991). "Standardized curriculum" mentioned in the Asian/American studies could be labeled by some educators as identical textbooks. Second-grade teachers were made aware of the NCTM Standards (1989) and second-grade outcomes. They taught to the same outcomes but were free to make decisions on what was important for reaching second-grade outcomes. Each teacher had chosen a methodology in which they felt most comfortable reaching those outcomes. Teaching to outcomes versus "standardized curriculum" needs to be investigated in the upper grades also.

This study used only one of multiple ways to assess mathematical achievement. Since evaluation is moving toward open questions and authentic assessments, the researcher would also recommend further study using a different dependent variable.

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Appendix A

Sample Questions of the Dependent Variable, the California Diagnostic Mathematics Assessment, and Objectives

California Diagnostic Mathematics Tests

Level B

---

1. Our ball team has 6 girls and 5 boys. How many children are on the team in all?

   ○ 5 - 6 = ☐

   ○ 6 + 5 = ☐

   ○ 6 + 6 = ☐

   ○ 6 - 5 = ☐

7. There were 17 apples in the basket. The children took 8 of them. How many apples are left?

8	17	17	8
+ 8	- 8	+ 8	- 8
○	○	○	○

10. Terry had 8 marbles. She found 7 more marbles on the playground. How many marbles does Terry have in all?

    ○ 8

    ○ 14

    ○ 15

    ○ 87

14. Abid read 60 pages of his book. His friend read 20 pages. How many pages did they read in all?

- ○ 40
- ○ 62
- ○ 80
- ○ 90

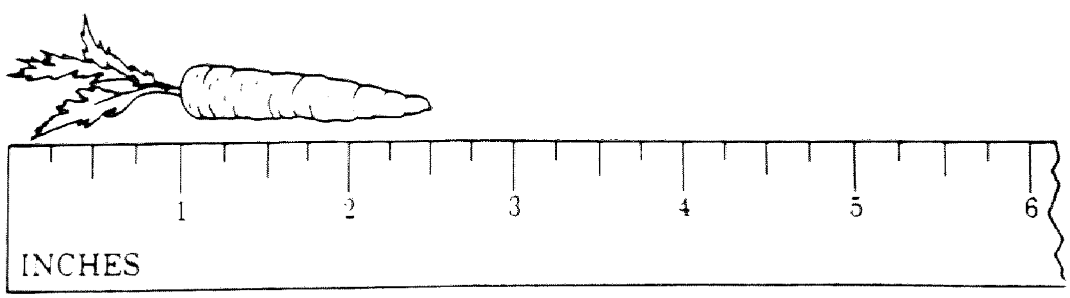
16. There were 70 trees at the tree farm. Then 15 more trees were planted. How many trees are there in all?

- ○ 95
- ○ 90
- ○ 85
- ○ 75

22. There were 70 green peppers in the garden. Jon picked 30 of them. How many green peppers are left in the garden?

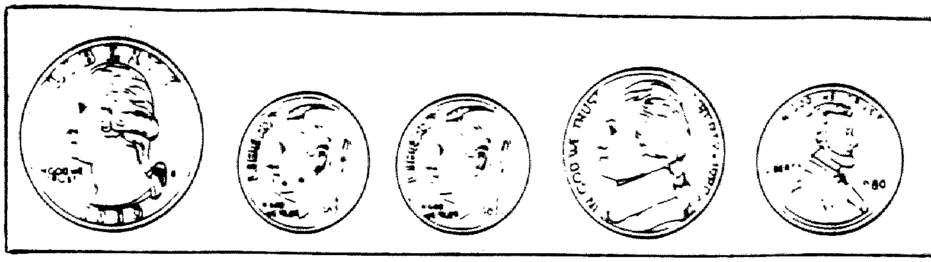
- ○ 20
- ○ 30
- ○ 40
- ○ 70

27. How long is the carrot?



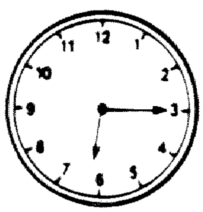
- ○ 2 inches
- ○ 2 1/2 inches
- ○ 3 inches
- ○ 3 1/2 inches

33. Count the money and fill in the correct bubble.



51¢	42¢	33¢	5¢
O	O	O	O

36. Fill in the correct bubble showing the time.



3:30	7:15	6:00	6:15
O	O	O	O

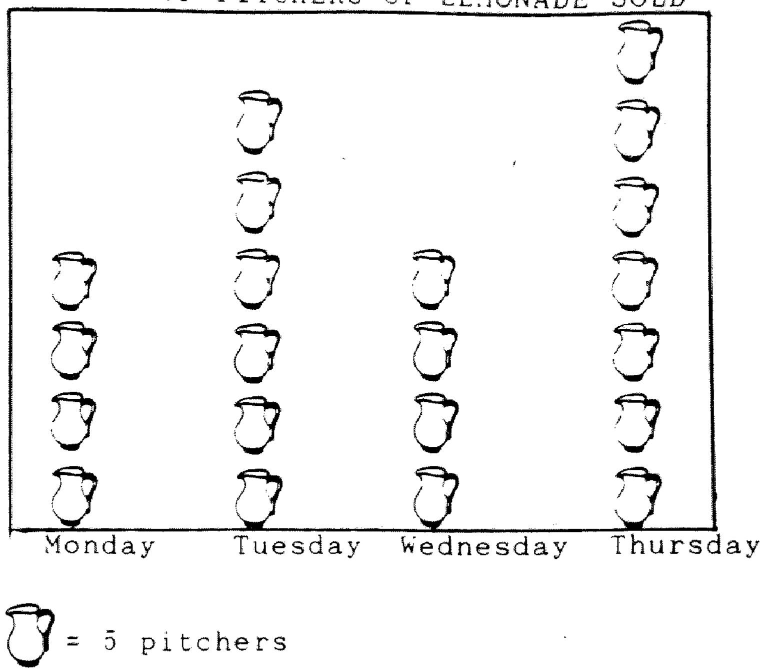
41. NUMBER OF STRAWBERRIES PICKED

Mike	Christy	Brian	Ross	Jackie
70	85	55	70	80

Who picked the most strawberries?

- O Jackie
- O Ross
- O Brian
- O Christy

NUMBER OF PITCHERS OF LEMONADE SOLD

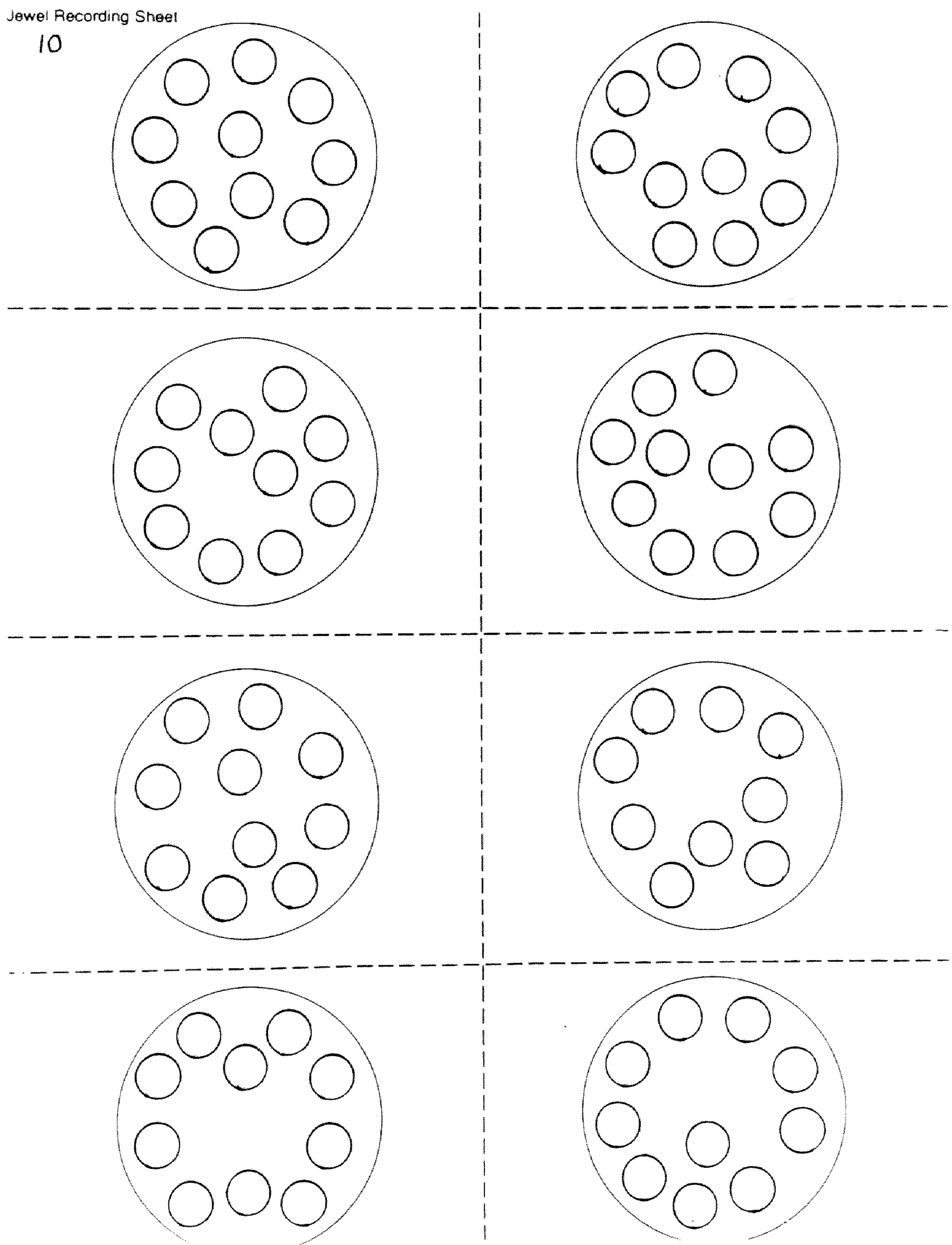


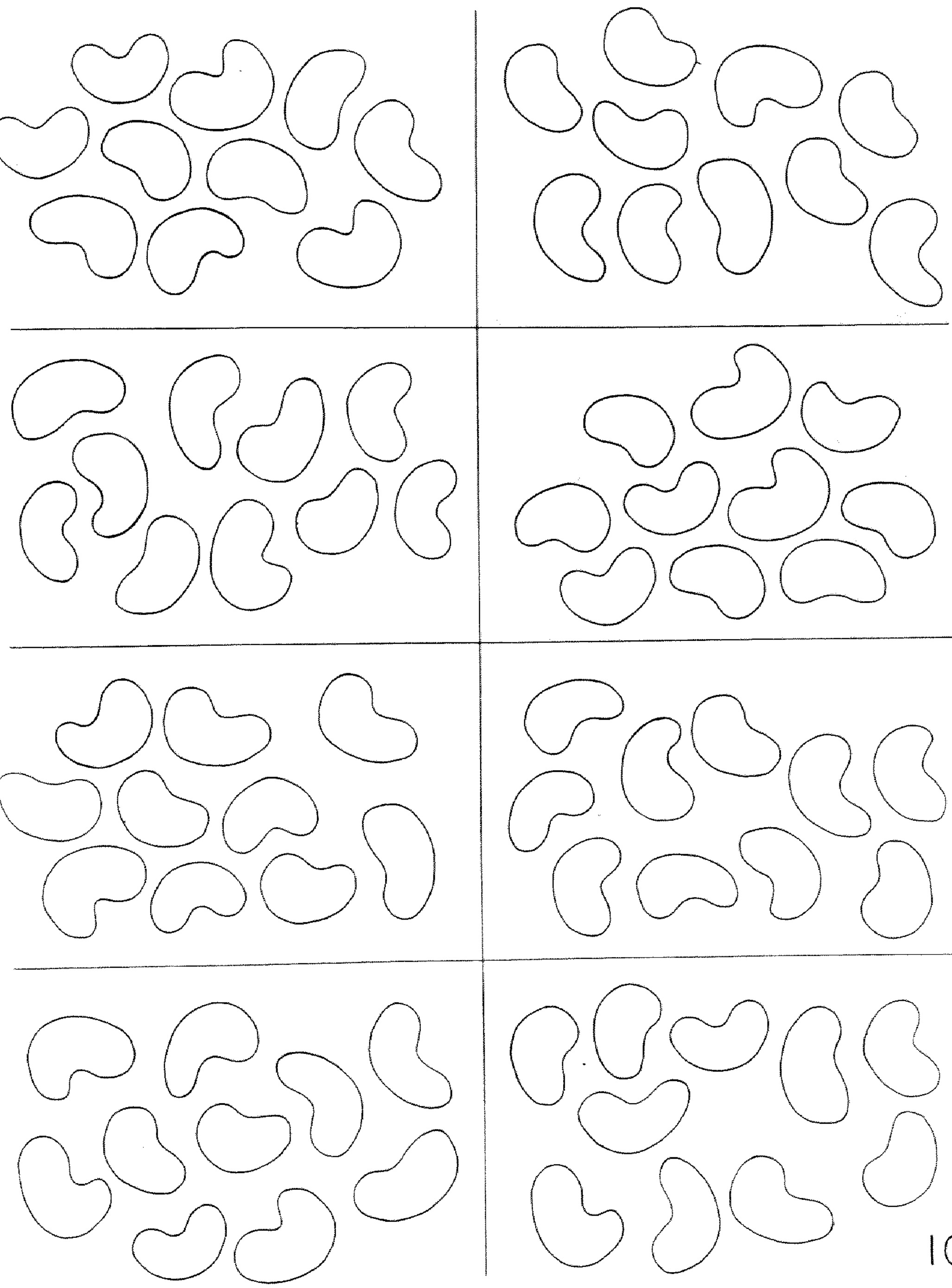
44. How many pitchers of lemonade were sold on Thursday?

- ○ 5
- ○ 7
- ○ 30
- ○ 35

Appendix B

Sample Student Worksheets Used in Treatment M




10

10

Name ______________________ **R4**

Add (+).

5 + 0 = 5 (///// ,)	3 + 1 (/// , /)	0 + 3 (, ///)
3 + 4 (/// , ////)	2 + 0 (// ,)	0 + 2 (, //)
0 + 5 (, /////)	2 + 6 (// , //////)	3 + 2 (/// , //)
8 + 0 (//////// ,)	5 + 1 (///// , /)	0 + 8 (, ////////)
4 + 2 (//// , //)	0 + 6 (, //////)	3 + 0 (/// ,)
0 + 4 (, ////)	7 + 0 (/////// ,)	1 + 4 (/ , ////)

Answering Problems

Write the answer to each problem.

A.	☐ ☐ ☐ ☐ ☐ ☐ = 6 ☒	☐ ☐ ☐ ☐ ☐ ☐ = ___ ☐ ☒	☐ ☐ ☐ ☐ ☐ ☐ = ___ ☐ ☐ ☒
	7 − 1 = 6	8 − 1 = ___	9 − 1 = ___
B.	☐ ☐ ☐ ☒ ☐ ☐ ☐ = ___ ☐ ☐ ☐	☐ ☐ ☒ ☐ ☐ ☒ = ___ ☐ ☐	☐ ☒ ☒ ☐ ☐ ☐ = ___ ☐ ☐ ☐
	10 − 1 = ___	8 − 2 = ___	9 − 2 = ___
C.	☐ ☐ ☒ ☒ ☐ ☐ ☐ = ___ ☐ ☐ ☐	☐ ☐ ☐ ☒ ☐ ☐ ☐ ☒ = ___ ☐ ☐ ☐	☒ ☒ ☒ ☐ ☐ ☐ = ___ ☐ ☐ ☐
	10 − 2 = ___	11 − 2 = ___	9 − 3 = ___
D.	☐ ☒ ☒ ☒ ☐ ☐ ☐ = ___ ☐ ☐ ☐	☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ = ___ ☒ ☒ ☒	☒ ☒ ☒ ☐ ☐ ☐ ☐ ☐ = ___ ☐ ☐ ☐ ☐
	10 − 3 = ___	11 − 3 = ___	12 − 3 = ___
E.	☒ ☒ ☒ ☒ ☐ ☐ ☐ = ___ ☐ ☐ ☐	☒ ☒ ☒ ☒ ☐ ☐ ☐ ☐ = ___ ☐ ☐ ☐	☒ ☒ ☒ ☒ ☐ ☐ ☐ ☐ = ___ ☐ ☐ ☐ ☐
	10 − 4 = ___	11 − 4 = ___	12 − 4 = ___

Missing Signs

Write the missing signs.

A. 8 + 2 = 10

B. 6 − 4 = 2

C. 2   7 = 9

D. 4   3 = 7

E. 7   3 = 4

F. 4   4 = 8

G. 9   1 = 10

H. 10   5 = 5

I. 9   2 = 7

J. 2   4 = 6

K. 8   3 = 5

L. 5   3 = 2

M. 5   5 = 10

N. 6   5 = 1

O. 9   3 = 6

P. 3   2 = 5

Q. 8   1 = 7

R. 4   5 = 9

S. 6   6 = 0

T. 7   2 = 5

U. 4   2 = 6

V. 3   5 = 8

W. 8   4 = 4

X. 1   6 = 7

Y. 7   2 = 9

Z. 10   6 = 4

Name____

Directions: Circle + or − to tell what you do.

1. Mary has 4 glasses and
   7 straws.
   How many more straws than
   glasses?

   \+ −

2. Beth has 15 cents.
   Jan has 7 cents.
   How much more does Beth
   have?

   \+ −

3. There are 6 apples and
   7 oranges on the table.
   How many in all?

   \+ −

4. Rosa caught 10 fish.
   Jerry caught 4 fish.
   How many more fish did
   Rosa catch?

   \+ −

5. 8 books are on the table.
   8 books are on the desk.
   How many books in all?

   \+ −

6. Alan had 12 peanuts.
   He gave away 9 of them.
   How many were left?

   \+ −

7. Duane saw 10 bears at the
   zoo. He saw 6 lions.
   How many animals did he
   see in all?

   \+ −

8. Spiders have 8 legs.
   Ants have 6 legs.
   How many more legs do
   spiders have?

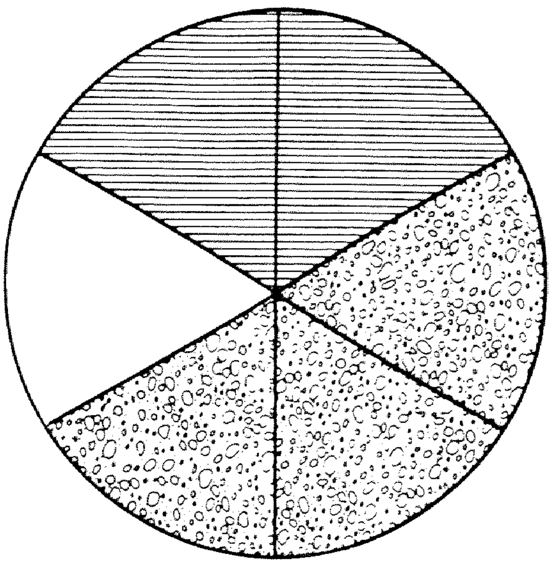
   \+ −

Name ______________________________

Sharing a Pizza **21**

Mark made a pizza for a spring picnic. He cut it into six pieces to share with his friends.

Key
Tiffany's slices
Mark's slices
Sam's slices



1. Who had the most slices of pizza? ______________
2. Who had only one piece of pizza? ______________
3. Who ate half of the pizza? ______________
4. How many pieces of pizza were there altogether? ______
5. Did everyone get the same amount of pizza? ______

Brainwork! Draw a circle graph like the one above. Color it to show that Mark, Sam, and Tiffany had the same number of pieces.

Name ______________ 13

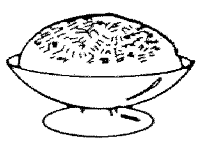
Make Your Own Sundae

CHICKABEE CORNERS ICE CREAM SHOP

ICE CREAM	TOPPINGS	
50¢	Chocolate Sauce	20¢
	Whipped Cream	15¢
	Sprinkles	10¢

Use the prices on the sign.
How much did each sundae cost?

1. Jonah made this sundae.



He spent ________¢.

2. Anita made this sundae.

She spent ________¢.

3. Carmen made this sundae.

He spent ________¢.

4. Elliot made this sundae.

He spent ________¢.

5. Art made this sundae.

He spent ________¢.

6. Wendy made this sundae.

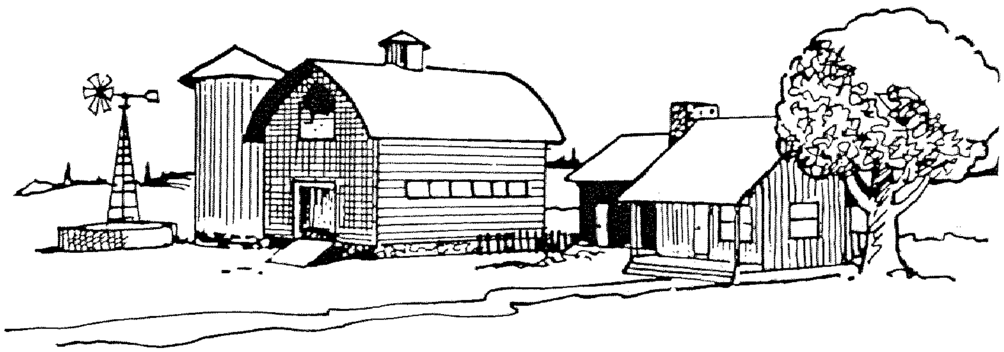
She spent ________¢.

6

Name ____________________

A Trip to the Chickabee Farm

Alice and Dan went to the Chickabee Farm.
They saw many animals.



Solve the problems.

1. There were 9 ducks swimming.

   There were 9 ducks walking on the grass.

   How many ducks were there altogether?

   ____

2. Alice saw 16 cows in the barn.

   She saw 7 goats in the field.

   How many more cows than goats did Alice see?

   ____

3. There were 12 horses and 6 ponies in the stable.

   How many horses and ponies were there in all?

   ____

4. Dan saw 12 pigs.

   9 pigs were black.

   How many pigs were not black?

   ____

5. 15 chickens were in the chicken

   10 chickens were in the farmyar

   How many chickens were there altogether?

   ____

6. There were 34 eggs in a basket

   Farmer McChick gave 12 eggs children.

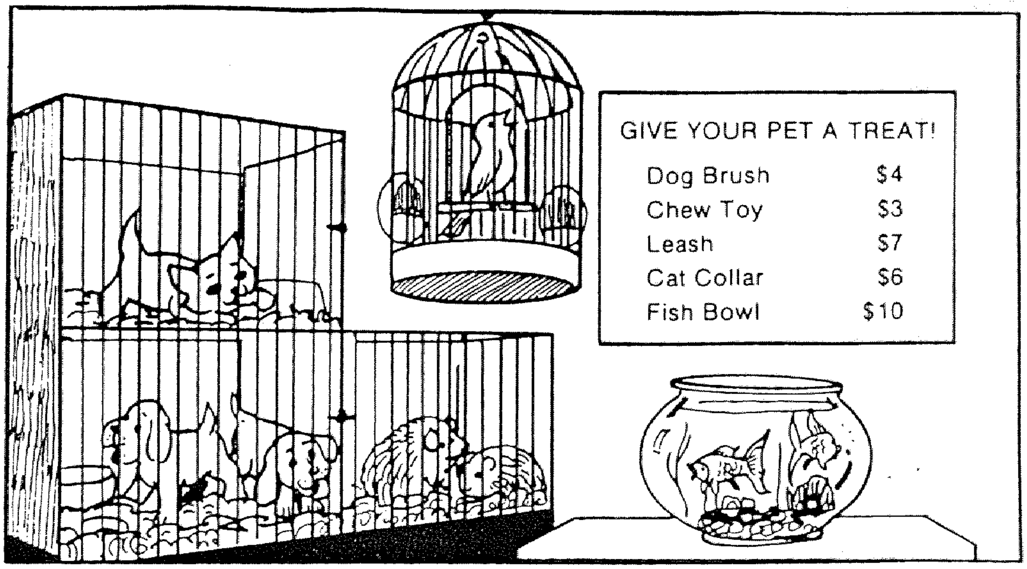
   How many eggs were left in the basket?

   ____

Name ______________

9

Dr. Doomuch's Pet Supplies



GIVE YOUR PET A TREAT!	
Dog Brush	$4
Chew Toy	$3
Leash	$7
Cat Collar	$6
Fish Bowl	$10

Use the facts in the sign.
Tell what each person bought.

1. Antonio bought 2 items.
   He paid $11.
   Antonio bought a ______________
   and a ______________.

2. Ana bought 2 items at the sale.
   She paid $16.
   Ana bought a ______________
   and a ______________.

3. Jud had $15.
   He bought one item at the sale.
   He has $5 left.
   Jud bought a ______________.

4. Mark had $10.
   He bought one item.
   He has $4 left.
   He bought a ______________.

5. Martha bought 3 different items.
   She paid $14.
   She bought a ______________,
   a ______________, and a
   ______________.

6. Laura spent $9 at the sale.
   She bought 3 items that were the same.
   Laura bought 3 ______________.

Appendix C

NCTM K-4 Standards and Summary of Changes in Content

K-4 STANDARDS

STANDARD 1: MATHEMATICS AS PROBLEM SOLVING

In grades K-4, the study of mathematics should emphasize problem solving so that students can:

- use problem-solving approaches to investigate and understand mathematical content;
- formulate problems from everyday and mathematical situations;
- develop and apply strategies to solve a wide variety of problems;
- verify and interpret results with respect to the original problem;
- acquire confidence in using mathematics meaningfully.

STANDARD 2: MATHEMATICS AS COMMUNICATION

In grades K-4, the study of mathematics should include numerous opportunities for communication so that students can:

- relate physical materials, pictures and diagrams to mathematical ideas;
- reflect upon and clarify their thinking about mathematical ideas and situations;
- relate their everyday language to mathematical language and symbols;
- realize that representing, discussing, listening, writing, and reading mathematics are a vital part of learning and using mathematics.

STANDARD 3: MATHEMATICS AS REASONING

In grades K-4, the study of mathematics should emphasize reasoning so that students can:

- draw logical conclusions about mathematics;
- use models, known facts, properties, and relationships to explain their thinking;
- justify their answers and solution processes;
- use patterns and relationships to analyze mathematical situations;
- believe that mathematics makes sense.

NCTM <u>Curriculum and Evaluation Standards for School Mathematics</u>, 1989, 23-69

K-4 STANDARDS

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- believe that mathematics makes sense.

NCTM Standards, (1989), 23-69

STANDARD 4: MATHEMATICAL CONNECTIONS
In grades K-4, the study of mathematics should include opportunities to make connections so that students can:
- link conceptual and procedural knowledge;
- relate various representations of concepts or procedures to one another;
- recognize relationships among different topics in mathematics;
- use mathematics in other curriculum areas;
- use mathematics in their daily lives.

STANDARD 5: ESTIMATION
In grades K-4, the curriculum should include estimation so students can:
- explore estimation strategies;
- recognize when an estimate is appropriate;
- use estimation to determine reasonableness of results;
- apply estimation in working with quantities, measurements, computation, and problem solving.

STANDARD 6: NUMBER SENSE AND NUMERATION
In grades K-4, the mathematics curriculum should include whole number concepts and skills so that students can:
- construct number meanings through real-world experiences and the use of physical materials;
- understand our numeration system by relating counting, grouping, and place-value concepts;
- develop number sense;
- interpret the multiple uses of numbers encountered in the real world.

STANDARD 7: PATTERNS AND RELATIONSHIPS
In grades K-4, the mathematics curriculum should include patterns and relationships so that students can:
- recognize, extend, describe, and create a wide variety of patterns;
- represent and describe mathematical relationships;
- explore the use of variables and open sentences to express relationships.

NCTM <u>Standards</u>, (1989), 23-69

SUMMARY OF CHANGES IN CONTENT AND EMPHASIS IN K-4 MATHEMATICS EDUCATION

INCREASED ATTENTION

NUMBER

- Number senses
- Place-value concepts
- Meaning of fractions and decimals
- Estimation of quantities

OPERATIONS AND COMPUTATION

- Meaning of operations
- Operational sense
- Mental computation
- Estimation and the reasonableness of answers
- Selection of an appropriate computational method
- Use of calculators for complex computation
- Thinking strategies for basic facts

GEOMETRY AND MEASUREMENT

- Properties of geometric figures
- Geometric relationships
- Spatial sense
- Process of measuring
- Concepts related to units of measurement
- Actual measuring
- Estimation of measurements
- Use of measurement and geometry ideas throughout the curriculum

PROBABILITY AND STATISTICS

- Collection and organization of data
- Exploration of chance

NCTM Standards, (1989), 20-21

PATTERNS AND RELATIONSHIPS

- Pattern recognition and description
- Use of variables to express relationships

PROBLEM SOLVING

- Word problems with a variety of structures
- Use of everyday problems
- Applications
- Study of patterns and relationships
- Problem-solving strategies

INSTRUCTIONAL PRACTICES

- Use of manipulative materials
- Cooperative work
- Discussion of mathematics
- Questioning
- Justification of thinking
- Writing about mathematics
- Problem-solving approach to instruction
- Content integration
- Use of calculators and computers

DECREASED ATTENTION

NUMBER

- Early attention to reading, writing, and ordering numbers symbolically

OPERATIONS AND COMPUTATION

- Complex paper-and-pencil computations
- Isolated treatment of paper-and-pencil computations
- Addition and subtraction without renaming
- Isolated treatment of division facts
- Long division
- Paper-and-pencil faction computation
- Use of rounding to estimate

NCTM Standards/ Grades K-04

NCTM <u>Standards</u>, (1989), 20-21

GEOMETRY AND MEASUREMENT

- Primary focus on naming geometric figures
- Memorization of equivalencies between units of measurement

PROBLEM SOLVING

- Use of clue words to determine which operation to use

INSTRUCTIONAL PRACTICES

- Rote practice
- Rote memorization of rules
- One answer and one method
- Use of worksheets
- Written practice
- Teaching by telling

NCTM Standards/ Grades K-4

NCTM Standards, (1989), 20-21

Appendix D

Second Grade Mathematics Outcomes and Student Evaluation

MATHEMATICS

GRADE 2

STUDENTS WILL.......

COUNTING
* Count from 0 to 100.
* Count from 83 to 127.
* Count from 375 to 425.
* Count backwards from 100.
* Count by 2's past 100.
* Count by 5's past 100.
* Count by 10's past 100.
* Count by 10's. Example - 15, 25, 35.

NUMBERS
* Write numbers through 99 correctly.
* Read three digit numbers.

ADDITION
* Know basic addition facts to 18.
* Add two digit numbers.
* Add two digit numbers with regrouping.
* Problem solve.

SUBTRACTION
* Know basic subtraction facts to 18.
* Subtract two digit numbers without regrouping.
* Subtract two digit numbers with regrouping.
* Problem solve.

MISSING NUMBERS
* Use manipulatives to complete number problems to 10.
  Example: 3 +   = 9
* Use manipulatives to complete number problems to 18.
  Example: 18 -   = 9

PLACE VALUE

* Identify ones, tens, and hundreds place when given a three digit number.
* Use manipulatives to build the number when given a three digit number.
* Apply concepts and use symbols of greater and less (< >) when given three digit number.

MONEY

* Identify the value and names of coins.
* Tell the amount of money shown in various sets of coins.
* Display a given amount of money.

TIME

* Tell time to five minute intervals.
* Problem solve involving time lapse.

MEASUREMENT

* Measure to the nearest inch.
* Measure to the nearest foot.
* Measure to the nearest centimeter.

Appendix E

Sample Student Worksheets Used in Treatment T

Use counters.
Add.

3.	4	1	2	5	0	4	5
	+ 4	+ 5	+ 7	+ 5	+ 8	+ 2	+ 4
	8						
4.	1	2	4	8	0	5	7
	+ 7	+ 3	+ 6	+ 1	+ 0	+ 2	+ 0
5.	0	7	3	2	5	0	3
	+ 5	+ 2	+ 1	+ 4	+ 1	+ 4	+ 6

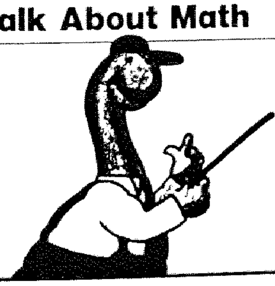
Notes for Home Children explore at the CONCRETE level using counters to find sums.

Subtract.

4.	13	13	5.	13	13	6.	14	14
	− 4	− 9		− 5	− 8		− 9	− 5
	9							

7.	14	12	11	12	13	11	14
	− 8	− 3	− 6	− 8	− 6	− 8	− 9
8.	11	13	11	12	11	13	14
	− 4	− 9	− 7	− 5	− 2	− 4	− 5
9.	12	13	13	11	14	12	13
	− 9	− 7	− 8	− 5	− 6	− 7	− 5
10.	12	11	11	12	14	13	14
	− 4	− 9	− 3	− 6	− 7	− 4	− 8

Talk About Math



Look at the facts below.
Why are they related facts?

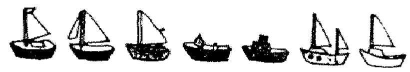
$13 - 5 = 8 \quad 13 - 8 = 5$

Notes for Home Children practice subtraction facts. Then they talk about related facts.

Name

PROBLEM-SOLVING STRATEGY

Choose an Operation

Fay has 7 boats. 

Ray has 4 boats. 

How many more boats does Fay have?

3 boats

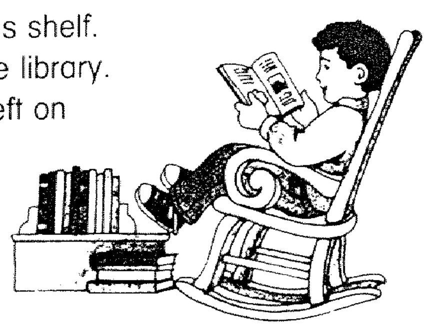
7
□ 4
3

Decide if you need to add or subtract.
Then solve.

1. Karen has 8 toy bears. She buys 7 more toy bears. How many toy bears does she have in all?

_____ bears

2. Jon has 15 books on his shelf. He takes 8 books to the library. How many books are left on the shelf?



_____ books

Notes for Home Children solve problems by deciding whether to add or subtract.

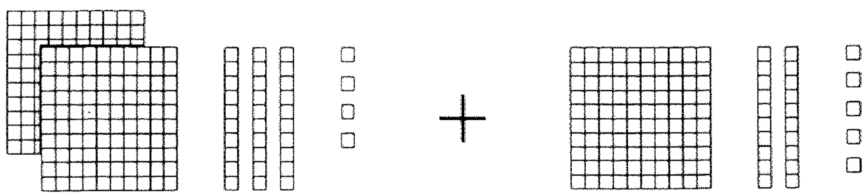
Exploring Math at Home

Dear Family,

In this chapter I have learned about three-digit addition and subtraction. I have also learned about money. Please help me with the activities below.

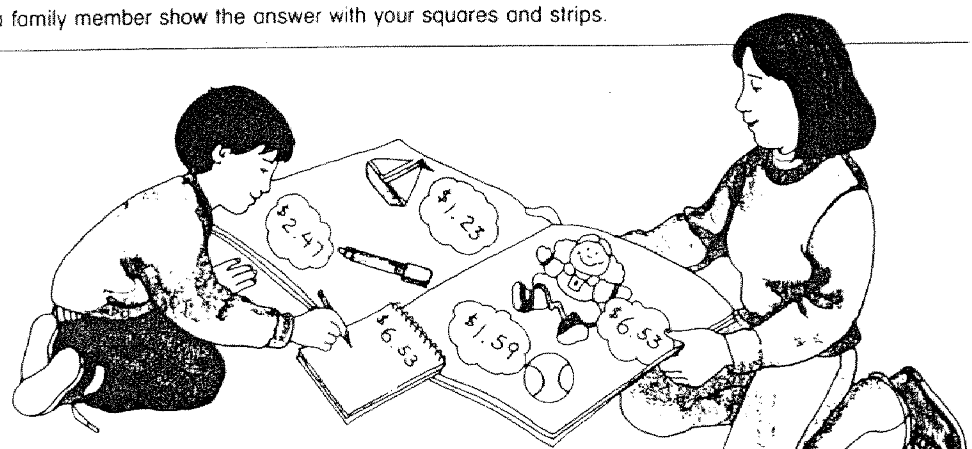
Love, ______________________

1.

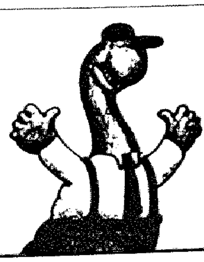


Use the squares and strips you used with Chapter 9. Show three-digit numbers. Add or subtract. Tell if you need to trade. Write the answer. Have a family member show the answer with your squares and strips.

2.



Use newspaper ads to find two objects with a combined cost of less than $10.00. Then find the difference between the costs of the same items. Your child should trade only once when adding or subtracting.



Coming Attractions

In the next chapter I will use counters and pictures to learn the meaning of multiplication and division.

Appendix F

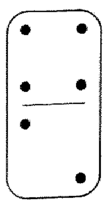
Sample Student Worksheets Used in Treatment C

Date ______________

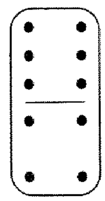
Domino Facts

Write 1 addition fact and 2 subtraction facts for each domino.

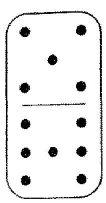
Unit



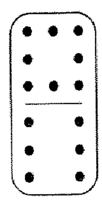
$\begin{array}{r} 4 \\ +\ 2 \\ \hline 6 \end{array}$ $\begin{array}{r} 6 \\ -\ 2 \\ \hline \end{array}$ $\begin{array}{r} 6 \\ -\ 4 \\ \hline \end{array}$



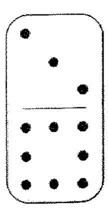
$\begin{array}{r} \square \\ +\ \square \\ \hline \end{array}$ $\begin{array}{r} \square \\ -\ \square \\ \hline \end{array}$ $\begin{array}{r} \square \\ -\ \square \\ \hline \end{array}$



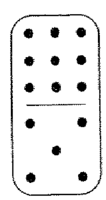
$\begin{array}{r} \square \\ +\ \square \\ \hline \end{array}$ $\begin{array}{r} \square \\ -\ \square \\ \hline \end{array}$ $\begin{array}{r} \square \\ -\ \square \\ \hline \end{array}$



$\begin{array}{r} \square \\ +\ \square \\ \hline \end{array}$ $\begin{array}{r} \square \\ -\ \square \\ \hline \end{array}$ $\begin{array}{r} \square \\ -\ \square \\ \hline \end{array}$

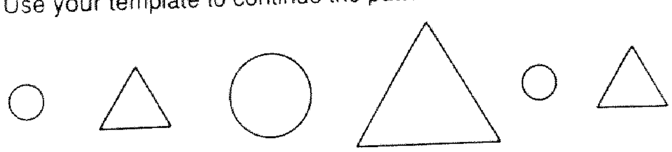


$\begin{array}{r} \square \\ +\ \square \\ \hline \end{array}$ $\begin{array}{r} \square \\ -\ \square \\ \hline \end{array}$ $\begin{array}{r} \square \\ -\ \square \\ \hline \end{array}$



$\begin{array}{r} \square \\ +\ \square \\ \hline \end{array}$ $\begin{array}{r} \square \\ -\ \square \\ \hline \end{array}$ $\begin{array}{r} \square \\ -\ \square \\ \hline \end{array}$

Use your template to continue the pattern.

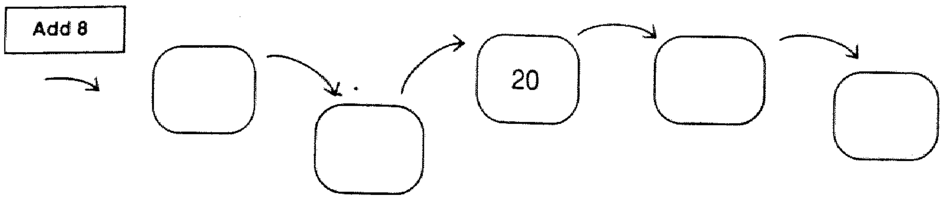


______________ ______________

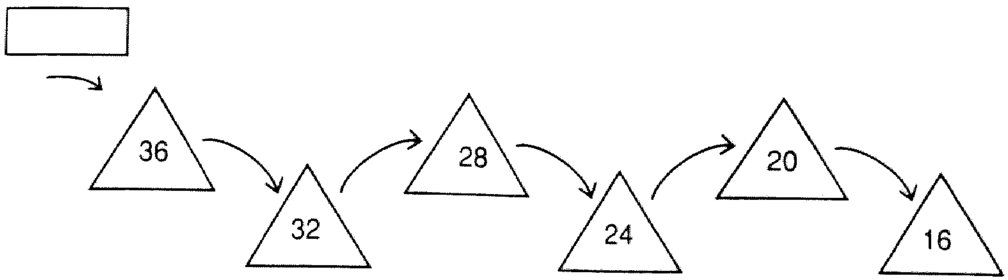
Date ____________

Frames and Arrows

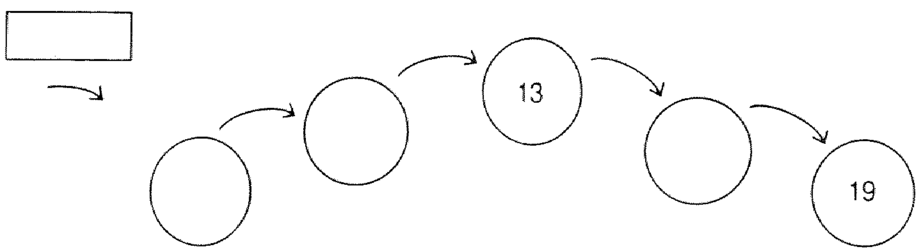
1. Fill in the frames.



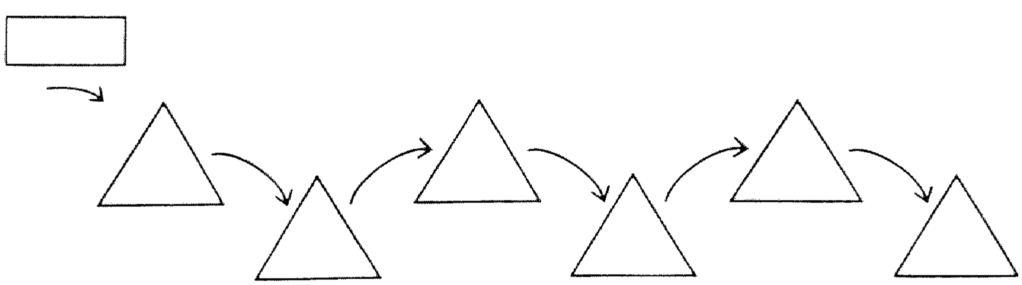
2. Fill in the arrow rule.



3. Fill in the arrow rule and the empty frames.



4. Make up your own problem. Ask your partner to solve it.

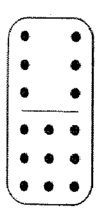


Date ____________

Addition Stories

Look at the Fruit and Vegetable Stand.
Make up some addition stories.
Record two of your stories.

Review Fact Families



$\square + \square$ $\square + \square$ $\square - \square$ $\square - \square$

Date ______________

Calculator Counts

1.

To count by 2
Press [2] and [+]. Then press [=] over and over.

Count by 2 on the calculator. Write the numbers.

2 4 6 ___ ___ ___ ___ ___ ___ ___

What number is added each time you press [=]? ___

2.

To count by 3
Press [3] and [+]. Then press [=] over and over.

Count by 3 on the calculator. Write the numbers.

3 6 ___ ___ ___ ___ ___ ___ ___ ___

What number is added each time you press [=]? ___

3.

To count by 7
Press [7] and [+]. Then press [=] over and over.

Count by 7 on the calculator. Write the numbers.

___ ___ ___ ___ ___ ___ ___ ___ ___ ___

What number is added each time you press [=]? ___

4. Make up your own.

Count by ___.

___ ___ ___ ___ ___ ___ ___ ___ ___ ___

What number is added each time you press [=]? ___

SKIP COUNTING PRACTICE
Part 1

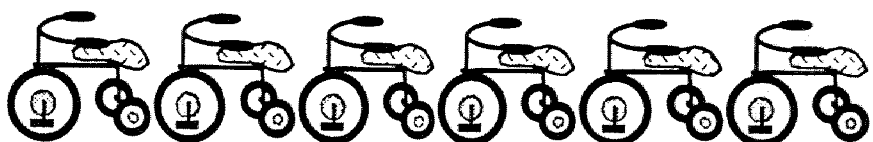
1. If one foot has 5 toes, how many toes would 4 feet have?

5 + 5 + 5 + 5

PRESS 5 (+) and (=) 4 times

4 feet have _______ toes.

2. If one tricycle has 3 wheels, how many wheels do 6 tricycles have?

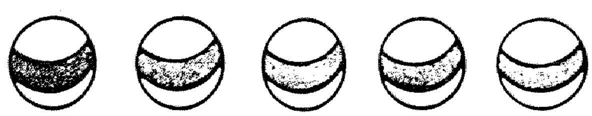


3 + 3 + 3 + 3 + 3 + 3

PRESS 3 (+) and (=) 6 times

6 tricycles have _______ wheels.

3. If one ball costs 2 dollars, how much would 5 balls cost?



2 + 2 + 2 + 2 + 2

PRESS 2 (+) and (=) 5 times

5 balls cost _______ dollars.

COUNTING ON

Use your calculator and fill in the blanks. Remember to CLEAR [C] after each problem.

5 [+] 1 PRESS [=] 3 times

_______ _______ _______

PRESS [C] to CLEAR

7 [+] 1 PRESS [=] 3 times _______ _______ _______

[C] to CLEAR

11 [+] 1 PRESS [=] 3 times _______ _______ _______

[C]

9 [+] 1 PRESS [=] 3 times _______ _______ _______

[C]

10 [+] 1 PRESS [=] 5 times _____ _____ _____ _____ _____

6 [+] 1 PRESS [=] 5 times _____ _____ _____ _____ _____

13 [+] 1 PRESS [=] 5 times _____ _____ _____ _____ _____

8 [+] 1 PRESS [=] 5 times _____ _____ _____ _____ _____

Use your calculator to find the three numbers that come **after** each number given. After you PRESS the number, remember to PRESS [+] 1 and [=] 3 times

17, ______, ______, ______

31, ______, ______, ______

26, ______, ______, ______

45, ______, ______, ______

COUNTING BACK

Use your calculator and fill in the blanks. Remember to CLEAR [C] after each problem.

6 [–] 1 PRESS [=] 3 times ________ ________ ________

12 [–] 1 PRESS [=] 3 times ________ ________ ________

25 [–] 1 PRESS [=] 3 times ________ ________ ________

33 [–] 1 PRESS [=] 3 times ________ ________ ________

16 [–] 1 PRESS [=] 3 times ________ ________ ________

9 [–] 1 PRESS [=] 3 times ________ ________ ________

62 [–] 1 PRESS [=] 3 times ________ ________ ________

27 [–] 1 PRESS [=] 3 times ________ ________ ________

Use your calculator to find the three numbers that come **before** each number given. After you PRESS the number, remember to PRESS [–] 1 and [=] 3 times.

21, ______, ______, ______

13, ______, ______, ______

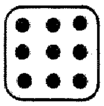
17, ______, ______, ______

40, ______, ______, ______

SUBTRACTION FACTS
Part 2

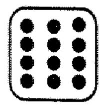
CLEAR

Count the dots. Use your calculator to subtract the number under each set of dots.

 = ____ dots

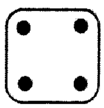
Subtract 1 dot.

____ - ____ = ____

 = ____ dots

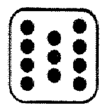
Subtract 2 dots.

____ - ____ = ____

 = ____ dots

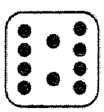
Subtract 3 dots.

____ - ____ = ____

 = ____ dots

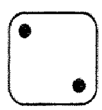
Subtract 4 dots.

____ - ____ = ____

 = ____ dots

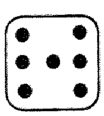
Subtract 7 dots.

____ - ____ = ____

 = ____ dots

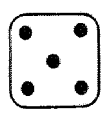
Subtract 1 dot.

____ - ____ = ____

 = ____ dots

Subtract 6 dots.

____ - ____ = ____

 = ____ dots

Subtract 3 dots.

____ - ____ = ____

ADDITION WORD PROBLEMS

1. A fisherman caught 5 fish one day. The next day he caught 3 fish. How many fish did he catch on both days? ______

2. A fisherman caught 12 fish one day. The next day he caught 18 fish. How many fish did he catch on both days? ______

3. Gerald had 6 toy cars. He got 4 more for his birthday. How many cars did he have then? ______

4. Gerald had 27 toy cars. He got 7 more for his birthday. How many did he have then? ______

5. A baker sold 3 cakes in the morning and 8 more in the afternoon. How many cakes did he sell that day? ______

6. A baker sold 19 cakes in the morning and 26 more in the afternoon. How many cakes did he sell that day? ______

7. There were 5 goldfish, 3 guppies, and 4 angelfish in an aquarium. How many fish were in the aquarium?______

8. There were 21 goldfish, 33 guppies, and 4 angelfish in an aquarium. How many fish were in the aquarium? ______

9. Michelle ate 8 grapes. Then she ate 7 more. How many grapes did she eat? ______

10. Michelle ate 28 grapes. Then she ate 35 more. How many grapes did she eat? ______
    Do you think she had a stomach ache? ______

FACT FAMILIES: ADDITION AND SUBTRACTION

Fill in the blanks with a + or − sign and an = sign. Do this in **two different ways. Answer as many as you can without your calculator.** Use your calculator to check your answers.

5 = 3 + 2 5 − 3 = 2	7 − 4 = 3 7 = 4 + 3	9 − 8 = 1 9 = 8 + 1
10 ____ 7 ____ 3 10 ____ 7 ____ 3	6 ____ 4 ____ 2 6 ____ 4 ____ 2	13 ____ 9 ____ 4 13 ____ 9 ____ 4
15 ____ 8 ____ 7 15 ____ 8 ____ 7	18 ____ 9 ____ 9 18 ____ 9 ____ 9	8 ____ 5 ____ 3 8 ____ 5 ____ 3
12 ____ 5 ____ 7 12 ____ 5 ____ 7	16 ____ 7 ____ 9 16 ____ 7 ____ 9	11 ____ 5 ____ 6 11 ____ 5 ____ 6
13 ____ 7 ____ 6 13 ____ 7 ____ 6	14 ____ 8 ____ 6 14 ____ 8 ____ 6	17 ____ 9 ____ 8 17 ____ 9 ____ 8

Appendix G

Second Grade Outcomes/Assessment/Report Card

SECOND GRADE MATH
STUDENT EVALUATION
FARMINGTON SCHOOL DISTRICT 192

STUDENT NAME______________________ DATE__________

M - Mastered T - Tested, but not mastered

TOPIC	STUDENT LEARNINGS	GOAL REACHED 1	2	3	4
I. COUNTING 1-8: 100% accuracy	1. Student can count from 0-100.				
	2. Student can count from 83-127.				
	3. Student can count from 375-425.				
	4. Student can count backwards from 100.				
	5. Student can count by 2's past 100.				
	6. Student can count by 5's past 100.				
	7. Student can count by 10's past 100.				
	8. Student can count by 10's (example - 15, 25, 35).				
II. NUMBERS 1 & 2: 100%	1. Student can write numbers through 99 correctly.				
	2. Student can read three digit numbers.				
III. ADDITION 1. 100 problems in 15 minutes: 95% 2. 10 problems: 90% 3. 10 problems: 90% 4. 10 story problems: 90%	1. Student knows basic addition facts to 18.				
	2. Student can add two digit numbers.				
	3. Student can add two digit numbers with regrouping.				
	4. Student can problem solve.				
IV. SUBTRACTION 1. 100 problems in 15 minutes: 95% 2. 10 problems: 90% 3. 10 problems: 90% 4. 10 story problems: 90%	1. Student knows basic subtraction facts to 18.				
	2. Student can subtract two digit numbers without regrouping.				
	3. Student can subtract two digit numbers with regrouping.				
	4. Student can problem solve.				

Skill	Criteria	Objectives	GOAL REACHED 1	2	3	4
V. **MISSING NUMBERS**	1 & 2. 10 problems: 90%	1. Using manipulatives, student can complete number problems to 10. (example 3 + = 9)				
		2. Using manipulatives, student can complete number problems to 18. (example 18 - = 9)				
VI. **PLACE VALUE**	1. 5 problems: 100%	1. Given a three digit number, student can identify ones, tens, and hundreds place.				
	2. 5 problems: 100%	2. Given a three digit number, student can use manipulatives to build the number.				
	3. 5 problems: 80%	3. Given three digit numbers, student can apply concepts and use symbols of greater and less than (< >).				
VII. **MONEY**	1. 5 problems: 100%	1. Student can identify the value and names of coins.				
	2. 5 problems: 80%	2. Student can tell the amount of money shown in various sets of coins, to a dollar				
	3. 5 problems: 80%	3. Student can display a given amount of money, to a dollar.				
VIII. **TIME**	1. 10 problems: 90%	1. Student can tell time to five minute intervals.				
	2. 5 problems: 80%	2. Student can do problem solving involving time lapse.				
IX. **MEASUREMENT**	1-3. 5 problems: 100%	1. Student can measure to the nearest inch.				
		2. Student can measure to the nearest foot.				
		3. Student can measure to the nearest centimeter.				

Appendix H

AGENDA FOR

ORIENTATION OF SECOND GRADE TEACHERS

SEPTEMBER 13, 1991

I. Schedule for pretest of California Diagnostic Mathematics Test:

Tuesday, September 17th

Jennifer Forst	9:05
Linda Roehl	10:15
Diana Kell	10:45
Louis Rutten	12:30
Marci Glassner	1:30

Wednesday, September 18th

Cindy Fees	9:15
Judy Sydness	10:00
Gina O'Brien	12:30
Marilyn Bah	1:30
Carolyn Nelson	2:30

II. The time block for mathematics will be the same for all second grade teachers.

III. Discussion on "teaching to the methodology" so as not to contaminate the study.

IV. a. Discussion of any additional resources we have available in mathematics so that we are using the same items for the same amount of time. For example, we came to an agreement on using <u>Arithmetic Developed Daily</u> (ADD) three times each week (story problems that were written by Racine, Wisconsin teachers mathematics.)

b. All second graders will recieve the same amount of computer time for mathematic drill and practice once every six days as scheduled.

V. Avoid discussion in the teachers lounge.

VI. Keep a daily journal as to what took place during mathematics time. Jot down ideas about whether students were:

1. Extremely successful.
2. Performed as expected.
3. Performance was below what was expected and you would not use again.

VII. Using the calendar, write down the concept covered each day in mathematics.

VIII. Keep track of the the times you had to reteach a concept.

IX. Teacher assistants will assist you in the quarterly assessment for the outcomes on the report card.

X. Hand in a copy of outcomes each quarter so I will know how many children have mastered the concepts that were assessed for the quarter.

XI. Our next meeting will be September 27th at 12:30.

XII. Posttest will be scheduled for the week of March 23.

XIII. Discussion of raw score results on our workshop day, April 1, 1992, before parent conferences.

Questionnaire

Name of Elementary School ______________________________

Address ______________________________

Name of the mathematics program used during the 91-92 school year:

( ) Scott Foresman "Exploring Mathematics": a traditional basal approach

( ) Mathematics Their Way plus worksheet packets designed by second grade teachers: A manipulative approach combined with a few symbolic level worksheets.

( ) University of Chicago Mathematics Project "Everyday Mathematics": A structured manipulative approach with calculators.

To what degree did you follow this curriculum?

( ) Carefully

( ) In combination with other materials but as primary curriculum.

( ) As a resource to other curriculum

How much time is spent each day on mathematics? ________min.

How many second graders do you have in your classroom? ____

How many hours a week does your classroom teacher assistant (Chapter I aide) assist in mathematics instruction?

______________________hours.

How many years of teaching experience in 2nd grade?

( ) Teacher for Treatment T: _____years experience

( ) Teacher for Treatment M: _____years experience

( ) Teacher for Treatment C: _____years experience

Do you have special training in mathematics education?

( ) Center of Innovation: week of staff development using manipulatives

( ) Staff development using calculators and hands on activities

( ) Any additional mathematics workshops on the graduate level beyond Bachelor of Science degree.

Indicate which occupational group best characterizes your school's parent population:

( ) business and professional

( ) industrial blue/collar

( ) both of the above

( ) other ________________(specify)

MEMO

TO: Second grade teachers

FROM: Margaret McKernan

RE: Instructions for Post-test for mathematics research

I'd like to conduct the applications portion of the California Diagnostic Mathematics post-test as I did in the fall. All of the students in all three treatments will be afforded exactly the same opportunities, time limits, directions, etc.

1. Tests will be given the week of March 16th with the following schedule:

   Monday, March 16th

Jennifer Forst	9:05
Linda Roehl	10:15
Diana Kell	10:45
Louis Rutten	12:30
Marci Glassner	1:30

   Tuesday, March 17th

Cindy Fees	9:15
Judy Sydness	10:00
Gina O'Brian	12:30
Marilyn Bah	1:30
Carolyn Nelson	2:30

2. Since experimental control is critical for valid research, the procedure will be exactly the same in all rooms. Please have the kids desks arranged so that children are unable to look at another student's answers.

3. Please remain in the classroom as you did in the fall and assist if a student has difficulty finding the bubble or a new page of the test.

Teacher Questionnaire for Second Grade Mathematics Research

1. Do you use quarterly evaluations for students? Yes No

2. Do you use tutorial programs for students? Yes No

3. Do you test students for mastery in skill areas using criterion-based tests? Yes No

4. Do your records include results of mastery tests? Yes No

5.. Do your students maintain separate record forms? Yes No

6. How often do you use ADD? (Arithmetic Developed Daily) Daily Biweekly Triweekly

7.. Do you meet with students individually to diagnose and prescribe for their specific needs? Often Occasionally Never

8. Did you use flash cards for mathematics facts? Daily Biweekly Triweekly

9.. Did you incorporate story problems into the lesson? Daily Biweekly Triweekly

10. Do you use time tests for facts? Often Occasionally Never

Teacher Observation Sheet

DATE______ TEACHER__________________ GROUP OBSERVED__________________ RM____

STUDENT

Concept Level	Connecting or Pictorial Level	Symbolic Level

TEACHER

Concrete Level	Connecting Level	Symbolic Level

DESCRIPTION OF ACTIVITY

RESOURCES USED

COMPLIANCE WITH NCTM STANDARDS

SECOND GRADE MATHEMATICS
CONCEPTS

SEPTEMBER

SUN	MON	TUES	WED	THUR	FRI	SAT
		17	18	19	20	
	23	24	25	26	27 Meet in Margaret's office at 12:30-3:30.	
	30 Return to Marget at the end of the month.					

Appendix I

Memo for Posttest and Follow-up Questionnaire

MEMO

DATE: March 2, 1992

TO: Second grade teachers

FROM: Margaret McKernan

RE: Instructions for Post-test for mathematics research

I'd like to conduct the applications portion of the California Diagnostic Mathematics post-test as I did in the fall. All of the students in all three treatments will be afforded exactly the same opportunities, time limits, directions, etc.

1. Tests will be given the week of March 16th with the following schedule:

Monday, March 16th

Jennifer Forst	9:05
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Marci Glassner	1:30

Tuesday, March 17th

Cindy Fees	9:15
Judy Sydness	10:00
Gina O'Brian	12:30
Marilyn Bah	1:30
Carolyn Nelson	2:30

2. Since experimental control is critical for valid research, the procedure will be exactly the same in all rooms. Please have the kids desks arranged so that children are unable to look at another student's answers.

3. Please remain in the classroom as you did in the fall and assist if a student has difficulty finding the bubble or a new page of the test.

Teacher Questionnaire for Second Grade Mathematics Research

1. Do you use quarterly evaluations for students?	Yes	No	
2. Do you use tutorial programs for students?	Yes	No	
3. Do you test students for mastery in skill areas using criterion-based tests?	Yes	No	
4. Do your records include results of mastery tests?	Yes	No	
5.. Do your students maintain separate record forms?	Yes	No	
6. How often do you use ADD? (Arithmetic Developed Daily)	Daily	Biweekly	Triweekly
7.. Do you meet with students individually to diagnose and prescribe for their specific needs?	Often	Occasionally	Never
8. Did you use flash cards for mathematics facts?	Daily	Biweekly	Triweekly
9.. Did you incorporate story problems into the lesson?	Daily	Biweekly	Triweekly
10. Do you use time tests for facts?	Often	Occasionally	Never

How often did you use co-operative learning in your mathematics lessons?

- ( ) Every day
- ( ) Tri weekly
- ( ) Bi weekly
- ( ) Weekly
- ( ) Once every two weeks

How many years of teaching experience in 2nd grade?

- ( ) Teacher for Treatment T: _____years experience
- ( ) Teacher for Treatment M: _____years experience
- ( ) Teacher for Treatment C: _____years experience

Do you have special training in mathematics education?

- ( ) Center of Innovation: week of staff development using manipulatives.
- ( ) Staff development using calculators and hands on activities.
- ( ) Any additional mathematics workshops on the graduate level beyond Bachelor of Science degree.

Indicate which occupational group best characterizes your school's parent population:

- ( ) Business and professional
- ( ) Industrial blue/collar
- ( ) Both of the above
- ( ) Other ______________________________(specify)

Appendix J

Tables of Raw Scores for Pretest and Posttest

RAW SCORES AND DATA FOR ANALYSIS OF COVARIANCE

Student #	Every student was identified by a #
School #:	1 FES Farmington Elementary School 2 ARES Akin Road Elementary School
Program #:	1 T - Traditional 2 M - Math Their Way 3 C - Chicago Math
Gender #:	1 M - Male 2 F - Female
Age:	The first number is the student's age in years. The second number refers to months.
Pretest } Posttest }	44 Problem solving/application questions on California Diagnostic Mathematics Test Level B by CTB McGraw Hill; intended for 2.6 - 3.9 grade levels.
IQ:	CSI or Cognitive Skills Index on Test of Cognitive Skills by CTB McGraw-Hill

Student	School	Treatment	Gender	Age	Pretest	Posttest	IQ
			ROOM #25				
1	1FES	1T	1T	8/1	34	43	115
2	1FES	1T	2F	7/7	37	41	108
3	1FES	1T	1M	8/6	40	43	95
4	1FES	1T	1M	7/10	32	43	124
5	1FES	1T	1M	8/2	31	44	123
6	1FES	1T	1M	8/0	20	35	99
7	1FES	1T	1M	8/5	40	44	115
8	1FES	1T	2F	7/9	32	41	138
9	1FES	1T	2F	8/1	36	Moved	
10	1FES	1T	1M	8/0	42	44	141
11	1FES	1T	2F	7/10	35	37	107
12	1FES	1T	1M	8/4	15	40	102
13	1FES	1T	1M	8/6	30	42	106
14	1FES	1T	2F	7/9	24	Moved	108
15	1FES	1T	1M	8/7	36	43	113
16	1FES	1T	2F	9/2	15	19	85
17	1FES	1T	1M	8/5	37	44	114
18	1FES	1T	2F	8/0	39	22	103
19	1FES	1T	1M	8/5	39	44	103
20	1FES	1T	2F	7/8	14	37	95
21	1FES	1T	2F	7/10	31	34	115
22	1FES	1T	2F	8/6	40	44	133
23	1FES	1T	1M	8/6	36	44	107
24	1FES	1T	1M	7/9	33	44	122
25	1FES	1T	1M	8/2	29	37	105

ROOM # 24

26	1FES	1T	1M	8/3	27	35	103
27	1FES	1T	1M	7/8	39	44	119
28	1FES	1T	1M	8/1	22	43	129
29	1FES	1T	1M	7/10	30	42	133
30	1FES	1T	1M	7/8	29	43	82
31	1FES	1T	2F	8/1	17	31	105
32	1FES	1T	2F		25	Moved	
33	1FES	1T	1M	8/2	20	36	93
34	1FES	1T	2F	7/7	32	40	140
35	1FES	1T	1M	7/8	31	44	134
36	1FES	1T	1M	8/1	16	26	101
37	1FES	1T	2F	7/9	17	40	130
38	1FES	1T	2F	8/1	32	40	89
39	1FES	1T	2F	8/4	21	40	103
40	1FES	1T	2F	8/4	26	41	110
41	1FES	1T	1M	8/2	21	32	126
42	1FES	1T	1M	8/1	36	43	124
43	1FES	1T	1M	8/4	43	44	126
44	1FES	1T	1M	7/11	44	44	141
45	1FES	1T	2F	7/10	26	38	115
46	1FES	1T	2F	8/10	30	37	109
47	1FES	1T	2F	7/10	31	38	112
48	1FES	1T	1M	7/11	32	39	120
49	1FES	1T	2F	8/4	26	40	104
50	1FES	1T	2F	8/0	43	43	132

ROOM # 20

51	1FES	1T	1M	7/9	22	39	94
52	1FES	1T	1M	8/6	40	44	121
53	1FES	1T	1M	7/9	22	36	102
54	1FES	1T	2F	8/2	37	44	88
55	1FES	1T	2F	7/10	19	26	101
56	1FES	1T	1M	9/0	23	42	124
57	1FES	1T	1M	8/5	20	25	103
58	1FES	1T	1M	8/1	34	43	123
59	1FES	1T	2F	8/6	22	31	99
60	1FES	1T	2F	8/0	6	30	83
61	1FES	1T	2F	7/7	27	41	93
62	1FES	1T	2F	8/2	29	40	128
63	1FES	1T	1M	8/3	26	40	94
64	1FES	1T	1M	7/10	28	Moved	119
65	1FES	1T	1M	8/7	31	40	99
66	1FES	1T	2F	8/4	20	31	93
67	1FES	1T	1M	8/5	25	41	123
68	1FES	1T	2F	8/1	14	Moved	81
69	1FES	1T	2F	8/0	26	39	104
70	1FES	1T	1M	8/0	17	43	97
71	1FES	1T	2F	8/0	37	40	84
72	1FES	1T	2F	8/1	28	38	87
73	1FES	1T	1M	7/8	17	24	88
74	1FES	1T	1M	8/5	35	41	95
75	1FES	1T	1M	9	36	43	112
76	1FES	1T	2F		33	Moved	

ROOM # 27

77	2ARES	2M	2F	8/3	28	32	87
78	2ARES	2M	2F	8/9	23	40	116
79	2ARES	2M	2F	9/3	33	43	134
80	2ARES	2M	2F	8/5	22	42	89
81	2ARES	2M	1M	8/6	23	42	102
82	2ARES	2M	2F	7/10	27	43	106
83	2ARES	2M	1M	8/3	43	43	141
84	2ARES	2M	2F	8/4	37	44	124
85	2ARES	2M	1M	8/3	27	43	95
86	2ARES	2M	2F	8/3	26	44	96
87	2ARES	2M	1M	7/6	40	44	134
88	2ARES	2M	2F	7/11	36	44	121
89	2ARES	2M	2F	8/6	13	36	75
90	2ARES	2M	1M	7/7	15	37	131
91	2ARES	1M	1M	8/1	39	43	118
92	2ARES	2M	2F	7/7	29	43	92
93	2ARES	2M	2F	8/3	37	44	93
94	2ARES	2M	1M	8/6	22	39	84
95	2ARES	2M	1M	8/7	41	44	108
96	2ARES	2M	1M	8/0	23	41	86
97	2ARES	2M	1M	7/10	24	43	124
98	2ARES	2M	1M	7/10	31	41	129
99	2ARES	2M	2F	8/6	40	41	141
100	2ARES	2M	2F	8/0	41	43	130
101	2ARES	2M	2F	8/1	38	42	110

ROOM # 25

102	2ARES	2M	1M	8/9	39	41	121
103	2ARES	2M	1M	8/6	21	33	104
104	2ARES	2M	2F	8/1	27	39	140
105	2ARES	2M	1M	7/8	25	43	132
106	2ARES	2M	1M	8/5	37	43	113
107	2ARES	2M	1M	8/5	35	44	112
108	2ARES	2M	2F	8/0	24	43	100
109	2ARES	2M	1M	8/3	19	40	104
110	2ARES	2M	2F	7/11	23	39	110
111	2ARES	2M	2F	8/1	11	44	79
112	2ARES	2M	1M	8/3	36	42	114
113	2ARES	2M	1M	8/9	31	39	102
114	2ARES	2M	1M	8/5	23	42	123
115	2ARES	2M	1M	8/1	23	40	128
116	2ARES	2M	1M	7/8	17	42	105
117	2ARES	2M	2F	7/10	13	39	115
118	2ARES	2M	1M	7/10	22	40	74
119	2ARES	2M	2F	8/3	22	36	110
120	2ARES	2M	2F	7/11	26	34	112
121	2ARES	2M	2F	7/7	16	33	119
122	2ARES	2M	2F	8/5	39	42	135
123	2ARES	2M	2F	7/8	24	39	128
124	2ARES	2M	2F	8/7	32	44	113
125	2ARES	2M	1M	8/10	30	42	109
126	2ARES	1M	1M	7/10	19	44	134
127	2ARES	2M	2F	7/9	21	40	137

ROOM # 42

128	2ARES	2M	2F	8/2	20	35	112
129	2ARES	2M	1M	8/6	23	30	125
130	2ARES	2M	1M	7/7	18	34	85
131	2ARES	2M	2F	7/10	11	36	125
132	2ARES	2M	2F	8/1	36	41	129
133	2ARES	2M	1M	8/6	21	35	108
134	2ARES	2M	2F	8/7	31	Moved	123
135	2ARES	2M	2F	7/10	21	25	98
136	2ARES	2M	1M	7/9	23	41	122
137	2ARES	2M	2F	8/3	31	42	106
138	2ARES	2M	2F	8/0	33	44	115
139	2ARES	2M	1M	8/0	16	20	105
140	2ARES	2M	2F	8/1	24	35	105
141	2ARES	2M	2F	7/11	26	39	106
142	2ARES	2M	1M	8/6	32	40	120
143	2ARES	2M	1M	8/0	12	27	118
144	2ARES	2M	1M	8/5	22	38	115
145	2ARES	2M	1M	7/8	17	29	121
146	2ARES	2M	1M	7/9	26	41	137
147	2ARES	2M	2F	8/7	14	15	58
148	2ARES	2M	1M	8/3	37	42	138
149	2ARES	2M	2F	7/9	31	42	116
150	2ARES	2M	2F	7/10	31	44	118
151	2ARES	2M	1M	7/11	43	44	127
152	2ARES	2M	2F	8/4	13	37	129

ROOM # 22

153	1FES	3C	1M	7/7	21	40	112
154	1FES	3C	2F	7/10	18	33	86
155	1FES	3C	2F	8/3	36	44	118
156	1FES	3C	2F	8/4	35	41	103
157	1FES	3C	1M	7/11	32	42	112
158	1FES	3C	1M	8/6	20	42	108
159	1FES	3C	2F	8/0	34	43	115
160	1FES	3C	2F	8/1	44	44	134
161	1FES	3C	1M	8/0	30	40	117
162	1FES	3C	1M	8/9	23	42	100
163	1FES	3C	1M	8/1	14	39	102
164	1FES	3C	2F	7/10	14	42	114
165	1FES	3C	1M	7/11	30	43	118
166	1FES	3C	1M	7/11	26	42	105
167	1FES	3C	1M	8/2	12	34	99
168	1FES	3C	1M	8/11	23	35	82
169	1FES	3C	2F	8/0	35	44	128
170	1FES	3C	2F	8/0	23	34	77
171	1FES	3C	2F	8/5	14	14	84
172	1FES	3C	1M	8/1	29	44	140
173	1FES	3C	1M	8/0	26	35	87
174	1FES	3C	1M	8/2	23	43	111
175	1FES	3C	2F	8/6	34	43	89
176	1FES	3C	1M	7/10	26	40	94
177	1FES	3C	2F	7/10	35	43	131

ROOM # 26

178	1FES	3C	1M	9/6	19	19	64
179	1FES	3C	2F	8/2	19	35	99
180	1FES	3C	1M	8/1	28	42	111
181	1FES	3C	1M	8/0	32	41	109
182	1FES	3C	1M	8/8	30	39	106
183	1FES	3C	2F	9/2	38	44	136
184	1FES	3C	2F	8/10	25	39	82
185	1FES	3C	1M	7/9	13	28	105
186	1FES	3C	1M	9/3	29	40	107
187	1FES	3C	1M	8/7	37	43	110
188	1FES	3C	2F	7/9	24	33	98
189	1FES	3C	2F	7/10	37	42	127
190	1FES	3C	1M	8/1	26	42	109
191	1FES	3C	2F	8/5	23	34	99
192	1FES	3C	2F	7/11	30	36	96
193	1FES	3C	1M	8/3	24	42	115
194	1FES	3C	2F	7/7	35	42	106
195	1FES	3C	2F	7/7	22	39	95
196	1FES	3C	2F	9/0	13	15	60
197	1FES	3C	1M	7/10	21	23	87
198	1FES	3C	1M	9/3	21	37	85
199	1FES	3C	1M	8/9	37	41	98
200	1FES	3C	1M	8/8	24	39	113
201	1FES	3C	1M	8/8	15	Moved	84

ROOM # 40

2ARES	3C	2F	8/4	32	42	102
2ARES	3C	1M	8/1	27	41	103
2ARES	3C	1M	8/6	11	26	89
2ARES	3C	2F	8/3	27	30	76
2ARES	3C	1M	7/9	12	31	95
2ARES	3C	2F	7/7	13	21	82
2ARES	3C	1M	7/10	23	35	87
2ARES	3C	2F	7/10	13	40	103
2ARES	3C	2F	7/10	10	34	126
2ARES	3C	1M	8/1	32	41	114
2ARES	3C	2F	8/0	15	39	108
2ARES	3C	1M	8/7	10	30	110
2ARES	3C	1M	7/11	17	36	110
2ARES	3C	2F	7/9	22	33	90
2ARES	3C	2F	8/0	22	41	73
2ARES	3C	1M	7/10	37	43	122
2ARES	3C	2F	8/3	43	43	115
2ARES	3C	2F		18	Moved	
2ARES	3C	1M	8/0	40	42	109
2ARES	3C	2F	7/9	24	39	93
2ARES	3C	1M	8/4	31	39	119
2ARES	3C	2F	7/9	13	34	97
2ARES	3C	1M	8/6	25	41	99
2ARES	3C	1M	8/1	29	37	101

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226	2ARES	3C	1M	7/9	40	43	98
227	2ARES	3C	2F	7/9	37	42	114
228	2ARES	3C	1M	8/4	18	38	97
229	2ARES	3C	2F	8/2	9	25	95
230	2ARES	3C	2F	7/9	33	43	127
231	2ARES	3C	2F	8/2	14	29	100
232	2ARES	3C	2F	8/3	35	43	88
233	2ARES	3C	2F	8/0	33	36	91
234	2ARES	3C	1M	7/11	29	40	101
235	2ARES	3C	1M	7/11	35	35	109
236	2ARES	3C	1M	8/5	43	42	97
237	2ARES	3C	2F	8/4	43	44	121
238	2ARES	3C	2F	9/4	10	21	87
239	2ARES	3C	1M	7/7	14	32	102
240	2ARES	3C	2F	8/2	24	38	85
241	2ARES	3C	1M	8/5	36	41	90
242	2ARES	3C	1M	8/9	33	42	98
243	2ARES	3C	1M	8/4	25	39	121
244	2ARES	3C	1M		12	Moved	
245	2ARES	3C	1M	8/8	28	38	98
246	2ARES	3C	1M	8/2	36	41	123
247	2ARES	3C	1M	7/10	18	34	90
248	2ARES	3C	2F	8/0	10	21	86
249	2ARES	3C	2F	8/1	18	36	97
250	2ARES	3C	2F	8/0	28	33	93

Appendix K

Analysis of Covariance Cell Values

Variable	Treatment T	Treatment M	Treatment C
Pretest	N = 70	N = 75	N = 95
	$\overline{X}$ = 28.957	$\overline{X}$ = 26.587	$\overline{X}$ = 25.694
	S = 8.489	S = 8.609	S = 9.199
	S^2 = 72.06	S^2 = 74.11	S^2 = 84.62
Posttest	N = 70	N = 75	N = 95
	$\overline{X}$ = 38.757	$\overline{X}$ = 39.24	$\overline{X}$ = 37.211
	S = 5.955	S = 5.616	S = 6.771
	S^2 = 31.54	S^2 = 31.54	S^2 = 45.85
I.Q.	N = 70	N = 75	N = 95
	$\overline{X}$ = 109.757	$\overline{X}$ = 113	$\overline{X}$ = 102.253
	S = 15.708	S = 17.554	S = 15.331
	S^2 = 246.74	S^2 = 308.14	S^2 = 235.03
Age	N = 70	N = 75	N = 95
	$\overline{X}$ = 97.754	$\overline{X}$ = 97.613	$\overline{X}$ = 98.116
	S = 4.393	S = 4.299	S = 5.029
	S^2 = 19.30	S^2 = 18.48	S^2 = 25.29
Gain Score	N = 70	N = 75	N = 95
	$\overline{X}$ = 9.8	$\overline{X}$ = 12.653	$\overline{X}$ = 11.516
	S = 6.891	S = 7.244	S = 6.714
	S^2 = 47.49	S^2 = 52.48	S^2 = 45.08

Appendix L

Other Results

Results by Treatment

Pretest	Posttest
1. No "big" difference in means in 3 treatments	1. No "big" differences in means in 3 treatments
2. No "big" difference in ranges in 3 treatments	2. 75% of students scored above 77% in all 3 treatments
	3. All 3 treatments had extremes on the low end

Results Overall

Pretest	Posttest
1. Extreme scores at both ends	1. Extremes at low ends
2. Mode of 23	2. 75% of students scored above 80%
3. 25% of students scored at 79% or better	3. Data skewed negatively

Results by Treatment

I.Q.	Age	Gain score
1. Treatment T had all students with an I.Q. above 80	1. No big difference in means between programs	1. Treatment M and Treatment C had "larger" gain scores than Treatment T
2. Treatment M and Treatment C had some students with I.Q.s in the 60s and 70s	2. All 3 treatments had at least 4 students older than 8.75 years	2. Treatment T had an extreme (-17)
3. Treatment M had 4 students below 80		
4. Treatment C had 5 students below 80		
5. Extremes in the low end in Treatment M and Treatment C		

Results Overall

I.Q.	Age	Gain score
1. 37 students below an I.Q. of 90	1. Range of 24 months	1. 9 students had no gain
2. Range of 83	2. 87% of students between 7 1/2 and 8 1/2 years old	2. 25% of students gained between 16 and 33 points from pretest to posttest
		3. Extremes at low end (negative gain)